

# Problem Set 4

September 15, 2009

1. Problem 1.40 in J&R.
2. Optimize the following:  $U = x^2 + 2x + 3z^2 - 6x + xz$  subject to  $2x + 2z = 32$
3. Assume that  $U = (x + 2)(y + 1)$ . No specific numerical values are assigned to the price and income parameters.
  - (a) Write the Lagrangian function
  - (b) Find the optimal  $x, y, \lambda$ .
  - (c) Find  $v(p_x, p_y, y)$
  - (d) Solve for  $P_x = 4, P_y = 6, M = 130$ . interpret their economic meaning.
4. Compute the Marshallian demand functions, indirect utility function, and expenditure function for the constant elasticity of substitution (CES) utility function  $U(x_1, x_2) = (x_1^r + x_2^r)^{1/r}$
5. Consider the utility function

$$U = 2x_1^{\frac{1}{2}} + 4x_2^{\frac{1}{2}} \quad (1)$$

- (a) Find the Marshallian demand functions for goods 1 and 2 as they depend on prices and income.
- (b) Find the Hicksian demand functions.
- (c) Find the expenditure function and verify that

$$\begin{aligned} x_1^H(p_1, p_2, U^0) &= \frac{\partial e(p_1, p_2, U^0)}{\partial p_1} \\ x_2^H(p_1, p_2, U^0) &= \frac{\partial e(p_1, p_2, U^0)}{\partial p_2} \end{aligned}$$

- (d) Find the indirect utility function and verify Roy's identity.

6. Consider the utility maximization problem

$$\text{Max } U(x_1, x_2) \text{ s.t. } p_1x_1 + p_2x_2 = 1 \quad (2)$$

where prices have been normalized by setting  $M = 1$ . Let  $U^*(p_1, p_2)$  be the indirect utility function and  $\lambda$  be the lagrange multiplier.

(a) Show that  $\lambda^M = \frac{\partial U^*}{\partial x_1} x_1^* + \frac{\partial U^*}{\partial x_2} x_2^*$ .

(b) Show that

$$\begin{aligned}\frac{\partial U^*}{\partial p_1} &= -\lambda^M x_1^* \\ \frac{\partial U^*}{\partial p_2} &= -\lambda^M x_2^*\end{aligned}$$

(c) Show that  $\lambda^M = -\left[\frac{\partial U^*}{\partial p_1} p_1 + \frac{\partial U^*}{\partial p_2} p_2\right]$

7. Consider the class of utility functions that are "additively separable", i.e.,

$$U(x_1, x_2) = U^1(x_1) + U^2(x_2) \quad (3)$$

(a) Find the first-order conditions for utility maximization for these utility functions. Show that diminishing marginal utility in at least one good is implied.

(b) Show that if there is diminishing marginal utility in each good then both goods are "normal", i.e., not inferior.

(c) Show that this specification does not imply

$$\frac{\partial x_i^M}{\partial p_j} = 0, \quad i \neq j \quad (4)$$

(d) Show, however, that if  $\frac{\partial x_i^M}{\partial p_j} = \frac{\partial x_j^M}{\partial p_i} = 0$  then  $U(x_1, x_2) = \alpha_1 \log x_1 + \alpha_2 \log x_2$ .

8.  $x^M(p, M)$  satisfies the following properties:

- It is hom0 in  $p$  and  $M$ .
- $\sum_i p_i x_i = M$

(a) For  $U(x_1, x_2) = kx_1^\alpha x_2^{1-\alpha}$ , where  $k > 0$ ,  $0 < \alpha < 1$ , verify the above properties.

9. For the utility function  $U(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$ , calculate the expenditure function.

10. Consider the utility function  $U(x_1, x_2, x_3) = (x_1 - b_1)^\alpha (x_2 - b_2)^\beta (x_3 - b_3)^\gamma$ .

(a) Derive the Hicksian demand and expenditure functions.

(b) Show that the derivatives of the expenditure function are the Hicksian demand function