Problem Set 8

November 13, 2008

1. Compute the Marshallian demand funcitons, indirect utility function, and expenditure function for the constant elasticity of substitution (CES) utility function $U(x_1, x_2) = (x_1^r + x_2^r)^{1/r}$

2. Consider the utility function
$$U = 2x_1^{\frac{1}{2}} + 4x_2^{\frac{1}{2}} \tag{1}$$
   (a) Find the Marshallian demand functions for goods 1 and 2 as they depend on prices and income.
   (b) Find the Hicksian demand functions.
   (c) Find the expdenditure function and verify that
$$\begin{aligned} x_1^H\left(p_1, p_2, U^0\right) &= \frac{\partial e\left(p_1, p_2, U^0\right)}{\partial p_1} \\ x_2^H\left(p_1, p_2, U^0\right) &= \frac{\partial e\left(p_1, p_2, U^0\right)}{\partial p_2} \end{aligned}$$
   (d) Find the indirect utility function and verify Roy's identity.

3. Consider the utility maximization problem
$$Max\ U\left(x_1, x_2\right) \text{ s.t. } p_1x_1 + p_2x_2 = 1 \tag{2}$$
   where prices have been normalized by setting $M = 1$. Let $U^*\left(p_1, p_2\right)$ be the indirect utility function and λ be the lagrange multiplier.
   (a) Show that $\lambda^M = \frac{\partial U^*}{\partial x_1}x_1^* + \frac{\partial U^*}{\partial x_2}x_2^*$.
   (b) Show that
$$\begin{aligned} \frac{\partial U^*}{\partial p_1} &= -\lambda^M x_1^* \\ \frac{\partial U^*}{\partial p_2} &= -\lambda^M x_2^* \end{aligned}$$

(c) Show that $\lambda^M = -\left[\frac{\partial U^*}{\partial p_1}p_1 + \frac{\partial U^*}{\partial p_2}p_2\right]$

(d) Prove that if $U(x_1, x_2)$ is hodr in (x_1, x_2) then $U^*(p_1, p_2)$ is hod(-r) in (p_1, p_2).

4. Consider the class of utility functions that are "additively separable", i.e.,

$$U(x_1, x_2) = U^1(x_1) + U^2(x_2) \tag{3}$$

(a) Find the first- and second-order conditions for utility maximization for these utility functions. Show that diminishing marginal utility in at least one good is implied.

(b) Show that if there is diminishing marginal utility in each good then both goods are "normal", ie., not inferior.

(c) Show that this specification does not imply

$$\frac{\partial x_i^M}{\partial p_j} = 0,\ i \neq j \tag{4}$$

(d) Show, however, that if $\frac{\partial x_i^M}{\partial p_j} = \frac{\partial x_j^M}{\partial p_i} = 0$ then $U(x_1, x_2) = \alpha_1 \log x_1 + \alpha_2 \log x_1$.

5. $x^M(p, M)$ satisifies the following properties:

- It is hod0 in p and M.
- $\sum_i p_i x_i = M$
- $x^M(p, M)$ is a convex set.

(a) For $U(x_1, x_2) = k x_1^{\alpha} x_2^{1-\alpha}$, where $k > 0$, $0 < \alpha < 1$, verify the above properties.

6. For the utility function $U(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$, calculate the expenditure function.

7. Consider the utility function $U(x_1, x_2, x_3) = (x_1 - b_1)^{\alpha}(x_2 - b_2)^{\beta}(x_3 - b_3)^{\gamma}$.

(a) Derive the Hicksian demand and expenditure functions.

(b) Show that the derivatives of the expenditure function are the Hicksian demand function

(c) Verify that the Slutsky equation holds.