Problem Set 7

October 30, 2008

1. Assume that the objective function f measures the net value of some activity and the constraint represents a restriction on some resource. Using $\phi_k = \lambda^*$, explain why the Lagrange multiplier imputes a shadow price to the resource, i.e., a marginal value fo that resource in terms of the objective specified in the model. Also, in these models, what can be said, if anything, about how this marginal evaluation of the resource changes as the constraint eases, i.e., as k increases?

2. Consider the production function $y = x_1^{\alpha_1} x_2^{\alpha_2}$.

   (a) Show that the constant-output factor demand functions have the form

   $$x_i^* = k_i w_i^{-\frac{\alpha_j}{\alpha_1+\alpha_2}} w_j^{\frac{\alpha_j}{\alpha_1+\alpha_2}} y^{\frac{1}{\alpha_1+\alpha_2}},\ i \neq j \tag{1}$$

   (b) Show that the cost function has the form

   $$C^* = (k_1 + k_2)\, w_1^{-\frac{\alpha_1}{\alpha_1+\alpha_2}} w_j^{\frac{\alpha_2}{\alpha_1+\alpha_2}} y^{\frac{1}{\alpha_1+\alpha_2}} \tag{2}$$

   (c) Show that

   $$\frac{\partial C^*}{\partial w_i} \neq x_i^* \tag{3}$$

3. What is the difference between the factor demand curves obtained from cost minimization and those obtained from profit maximization? What observable (in principle) differences are there between the two?

4. If the law of diminishing returns applies to both factors, show that the factors are technical complements; i.e,., the marginal product of either factor rises when more of the other factor is applied.

5. Show that if the marginal products are positive, the isoquants must be downward-sloping.