

Problem Set 2

September 5, 2008

1. Analyze the logical forms of the following statements.

- (a) If anyone in the dorm has the measles, then everyone who has a friend in the dorm will have to be quarantined.
- (b) Jane saw a bear, and Roger saw one too.
- (c) Every number that is larger than x is larger than y .
- (d) If there is a number x such that $x^2 + 5x = w$ and there is a number y such that $4 - y^2 = w$, then w is between -10 and 10 .
- (e) Anyone who has bought a Rolls Royce with cash must have a rich uncle.
- (f) If nobody failed then test, then everybody who got an A will tutor someone who got a D.

2. Consider the following incorrect theorem:

Theorem 1 *Suppose n is a natural number larger than 2 and n is not a prime number. Then $2n + 13$ is not a prime number.*

- (a) What are the hypotheses and conclusion of this theorem?
- (b) Show that the theorem is incorrect by finding a counterexample.

3. Suppose $A \setminus B \subseteq C \cap D$ and $x \in A$. Prove that if $x \notin D$ then $x \in B$.

4. Suppose x is a real number and $x \neq 0$. Prove that if $\frac{\sqrt[3]{x+5}}{x^2+6} = \frac{1}{x}$ then $x \neq 8$.

5. Suppose a, b, c , and d are real numbers, $0 < a < b$ and $d > 0$. Prove that if $ac \geq bd$ then $c > d$.

6. Suppose x and y are real numbers and $3x + 2y \leq 5$. Prove that if $x > 1$ then $y < 1$.

7. Consider the following theorem.

Theorem 2 *Suppose x is a real number and $x \neq 4$. If $\frac{2x-5}{x-4} = 3$ then $x = 7$.*

(a) What wrong with the following proof of the theorem?

Proof. Suppose $x = 7$. Then $\frac{2x-5}{x-4} = \frac{2(7)-5}{7-4} = \frac{9}{3} = 3$. Therefore if $\frac{2x-5}{x-4} = 3$ then $x = 7$. ■

(b) Give a correct proof of the theorem.

8. Suppose $A \subseteq C$ and B and C are disjoint. Prove that if $x \in A$ then $x \notin B$.

9. Use proof by contradiction to prove the following theorem.

Theorem 3 Suppose $A \cap C \subseteq B$ and $a \in C$. Prove that $a \notin A \setminus B$.

10. Use proof by contradiction to prove the following theorem.

Theorem 4 Suppose $A \subseteq B$, $a \in A$, and a and b are not both elements of B . Prove that $b \notin B$.

11. Consider the following incorrect theorem.

Theorem 5 Suppose x and y are real numbers and $x + y = 10$. Then $x \neq 3$ and $y \neq 8$.

(a) What is wrong with the following proof of the theorem?

Proof. Suppose the conclusion of the theorem is false. Then $x = 3$ and $y = 8$. But then $x + y = 11$, which contradicts the given information that $x + y = 10$. Therefore, the conclusion must be true. ■

(b) Show that the theorem is incorrect by finding a counterexample.

12. Prove that if A and $B \setminus C$ are disjoint, then $A \cap B \subseteq C$

13. Suppose x is a real number.

(a) Prove that if $x \neq 1$ then there is a real number y such that $\frac{y+1}{y-2} = x$

(b) Prove that if there is a real number y such that $\frac{y+1}{y-2} = x$ then $x \neq 1$.

14. Prove that for every real number x , if $x > 2$ then there is a real number y such that $y + \frac{1}{y} = x$

15. Prove that if $A \subseteq B$ and $A \subseteq C$ then $A \subseteq B \cap C$

16. Suppose $A \subseteq B$. Prove that for every set C , $C \setminus B \subseteq C \setminus A$.

17. Prove

(a) A sufficient condition for the demand for a good to increase when its price falls is that it is a normal good.

(b) A necessary but not sufficient condition for the demand for a good to decrease when its price falls is that it is an inferior good.