

**Midterm**  
Econ 504

For credit, you must show your work.

Name \_\_\_\_\_

1. (a) Use a truth table to determine whether the following is a valid argument.

I will buy a new goat or a used Yugo. If I buy both a new goat and a used Yugo, I will need a loan. I bought a used Yugo and I don't need a loan. Therefore, I didn't buy a new goat.

- (b) Write the following in logic notation: 'All best friends chat'.

2. Prove that if  $u(x_1, x_2)$  and  $v(x_1, x_2)$  are both homogeneous of degree  $r$ , then  $s(x_1, x_2) = u(x_1, x_2) + v(x_1, x_2)$  is homogeneous of degree  $r$ .

3. Suppose  $f(x_1, x_2) = \sqrt{x_1^2 + x_2^2}$ .

(a) Show that  $f(x_1, x_2)$  is homogeneous of degree 1.

(b) Verify Euler's Theorem for this example.

4. Consider a profit-maximizing firm with production function  $y = f(x_1, x_2)$  that sells its output at a competitive price  $p$ . The firm obtains input  $x_1$  at a competitively determined unit wage  $w_1$ .  $w_2 = w_2^0 + kx_2$ .  $w_1$ ,  $w_2^0$ ,  $k$ , and  $p$  are all positive parameters.

(a) Calculate the first and second order conditions that are required for a maximum.

(b) Explain the general intuition behind the first and second order conditions.

(c) Derive the the impact of a change in  $w_1$  on  $x_1$  and  $x_2$ . Interpret your result.

5. At a conference seminar, a delegate says that he has estimated that in his study, the utility function for women is  $U_w = 2x_1^2x_2^2$  and the utility function for men is  $U_m = 3\log(x_1) + 3\log(x_2)$ . Based on these results he concludes that men and women have fundamentally different preferences. Write a paragraph that explains whether you agree or disagree with this conclusion. Support your explanation mathematically.