Profit Maximization

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1 Profit Maximization

To make things simple, we assume (for the moment) that firms are run by 1 powerful manager with dictatorial power. By assuming away the problem of infighting and office politics, we can make the assumption that the entire firm is dedicated on one goal: profit maximization. Recall,

$$\pi = R(q) - TC(q)$$

where $R(q)$ is revenue and $TC(q)$ is total costs. Lets first think of the problem of maximizing profits graphically and then use this to motivate the mathematical approach. The top graph below shows the revenue and total cost functions. The difference between the two functions is profit, which is also shown in the second graph. At output less than q_1 and output greater than q_2, total cost is greater than revenue so profits are negative. At q_1 and q_2, revenue equals costs, so profits are zero. At $q_1 < q < q_2$, revenue is greater than costs, so profits are positive. If we want to maximize profits, we want to pick the output level q at which the difference between revenues and cost is the greatest. This occurs at q^*. Note that for the second graph, since we have a nice hill shape, the maximum profit occurs where the slope of the profit function is 0. In other words, it occurs where $\frac{d\pi}{dq} = 0$. From our equation above, this means that at q^*,

$$\begin{aligned} \frac{d\pi}{dq} &= R'(q) - TC'(q) = 0 \\ R'(q) &= TC'(q) \\ MR &= MC \end{aligned}$$

When we are profit maximizing, marginal revenue (MR) equals marginal cost (MC). How can we see this on the first graph? MR is the slope of the revenue function. MC is the slope of the total cost function. So at q^*, the slopes of the two functions should be the same. Aha, they are.

1.1 Interpretation of the profit maximization condition

MR is the extra revenue you would earn from producing one more unit. MC is the extra cost of from producing one more unit. The profit maximization condition, $MR = MC$, says that when you are maximizing profit, the extra revenue you would get from producing one more unit exactly equals the extra cost of producing one more unit. To understand why this must be true, consider what would happen if this condition did not hold. Suppose $MR > MC$. This says that if you produced one more unit, you would gain more in revenue than it would cost you to produce the good. Obviously it would make sense to produce the good as you could increase your profit. So anywhere where $MR > MC$ can't possibly be profit maximizing. Now suppose $MR < MC$. This says that if you produced one less unit, your costs would fall more than your revenue would. Obviously it would make sense to produce one less good as it would increase your profit. So anywhere where $MR < MC$ can't possibly be profit maximizing. That leaves the only place that is an equilibrium, the only place where you could not change your output and increase your profits, as the output level at which $MR = MC$.

2 Marginal Revenue

As noted earlier, marginal revenue is the extra revenue from selling one more unit. We know that MR can be represented mathematically as the derivative of the revenue function with respect to quantity.

$$MR = \frac{Rev}{dq}$$

We know that revenue is price times quantity.

2.1 Price taker

If we assume that a firm is a price-taker, meaning that regardless of how much it produces, it has no effect on price, then revenue is

$$Rev = pq.$$

In this case,

$$MR = p.$$

Intutively, this says that the additional revenue one gets from producing an additional unit of output is just a constant price when how much you produce has no effect on price. This equation tells us that MR is a constant function and equal to the market price.

2.2 Price Maker

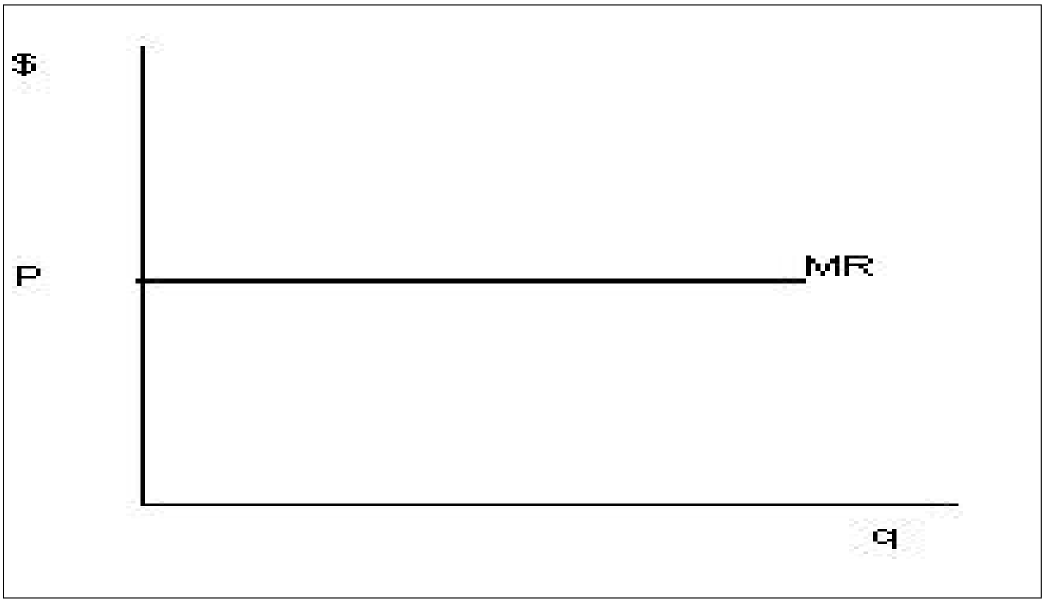
Now suppose that a firm is a price maker; how much a firm produces affects the price it receives. Therefore,

$$Rev = p(q)\,q.$$

This equation simply says that the output level affects the price; it isn't a constant. We calculate MR in the same way as above.

$$\begin{aligned} MR &= \frac{Rev}{dq} \\ &= p'(q)\,q + p \end{aligned}$$

Figure 1: MR of price taker



Now MR is a constant p term plus a second term, $p'(q)\,q$. We know that $q > 0$ and $p'(q) < 0$ by law of demand (for a non-Giffen good). Therefore, we can infer that $MR < p$ for a price-maker. Producing an additional unit increases a firm's revenue by less than the price at which it sells the good. This occurs because of the combination of the price and output effects. The second term is the output effect. The direct effect of selling an additional unit is the price you get. The first term in the above equation is the price effect. In order to sell an additional unit, you need to decrease the price. This loss in price multiplied by the original number of units you were selling is a decrease in revenue.

3 The Profit Maximization Problem

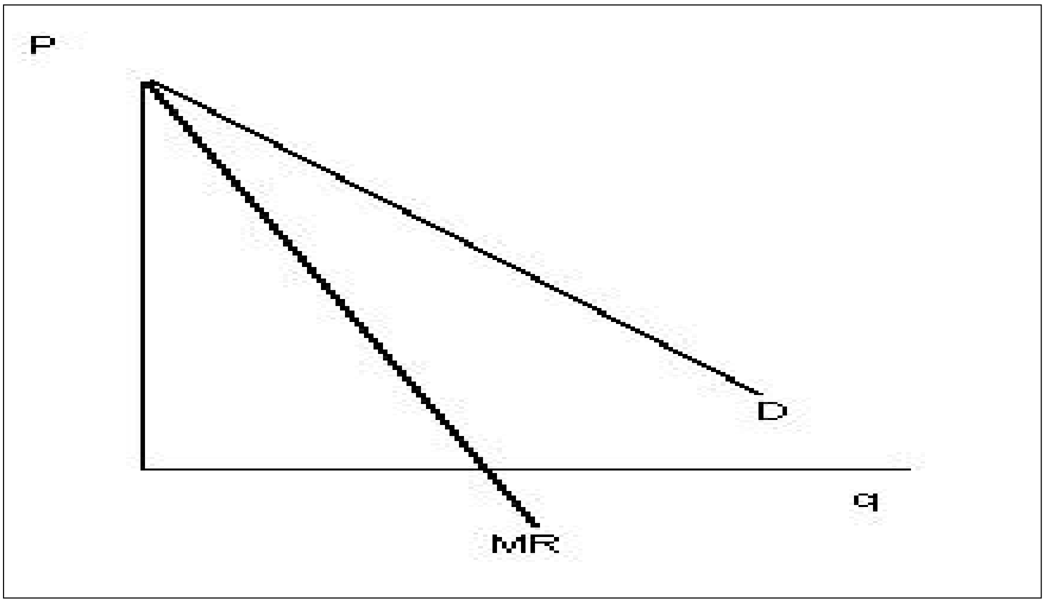
The firm's problem is to choose the level of output that will maximize output.

3.1 Price Taker

For the moment, we assume that the firm is a price taker. Regardless of how much the firm produces, it has no affect on price. Therefore, the firm's revenue is $p * q$ and it's problem is:

$$\begin{aligned} & \underset{q}{Max}\ pq - TC(q) \\ p'(q)\,q + p - MC &= 0 \\ MC &= p'(q)\,q + p \\ MC &= MR \end{aligned}$$

Figure 2: MR of price maker



Note that this is the same result as above: the profit maximizing quantity is one where $MC = MR$. As is shown in the graph below, the profit maximizing quantity, q^*, occurs where $MR = MC$.

3.2 Price Maker

Now assume that the firm is a price maker. How much the firm produces affects price. Therefore, the firm's problem is:

$$\underset{q}{Max}\ p(q)\,q - TC(q)$$

$$\begin{aligned} p - MC &= 0 \\ MC &= p \end{aligned}$$

Note that this is the same result as above: the profit maximizing quantity is one where $MC = MR$. In this case, $R(q) = pq$ so $MR = p$. As is shown in the graph below, the profit maximizing quantity, q^*, occurs where $MR = MC$.

4 Supply Decision of Price-Taker

$$\begin{aligned} \pi &= pq^* - STC(q^*) \\ &= q^*\left(p - \frac{STC(q^*)}{q^*}\right) \\ &= q^*\left(p - SAC(q^*)\right) \end{aligned}$$

Figure 3: Profit Maximization of price taker

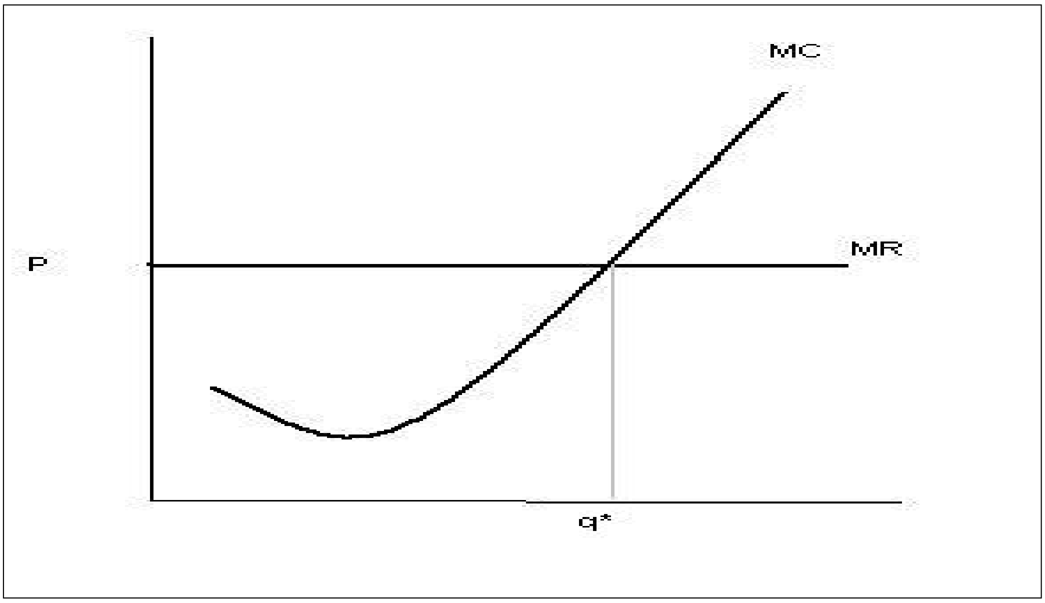
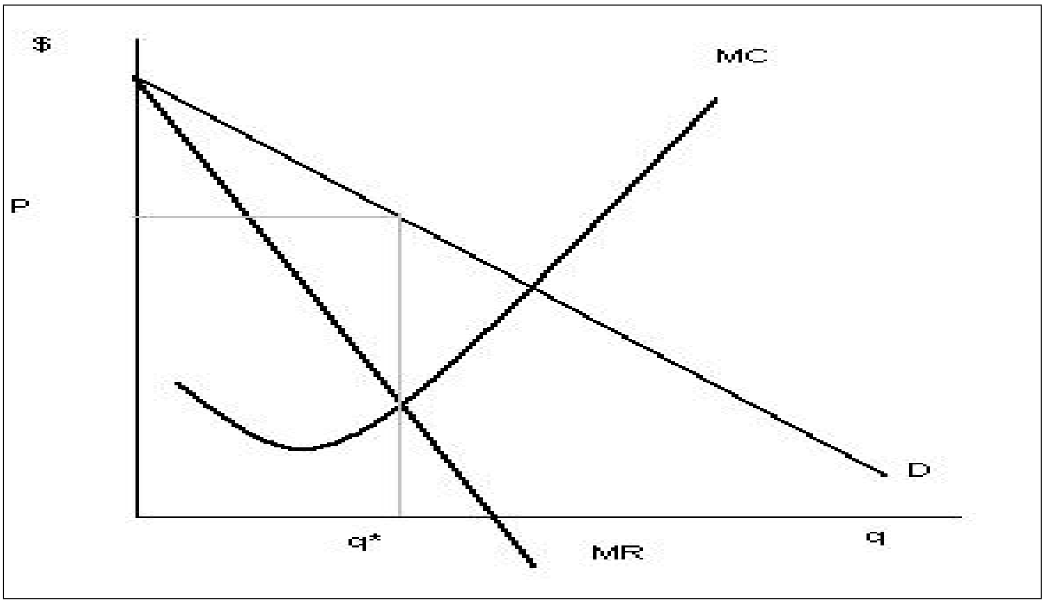
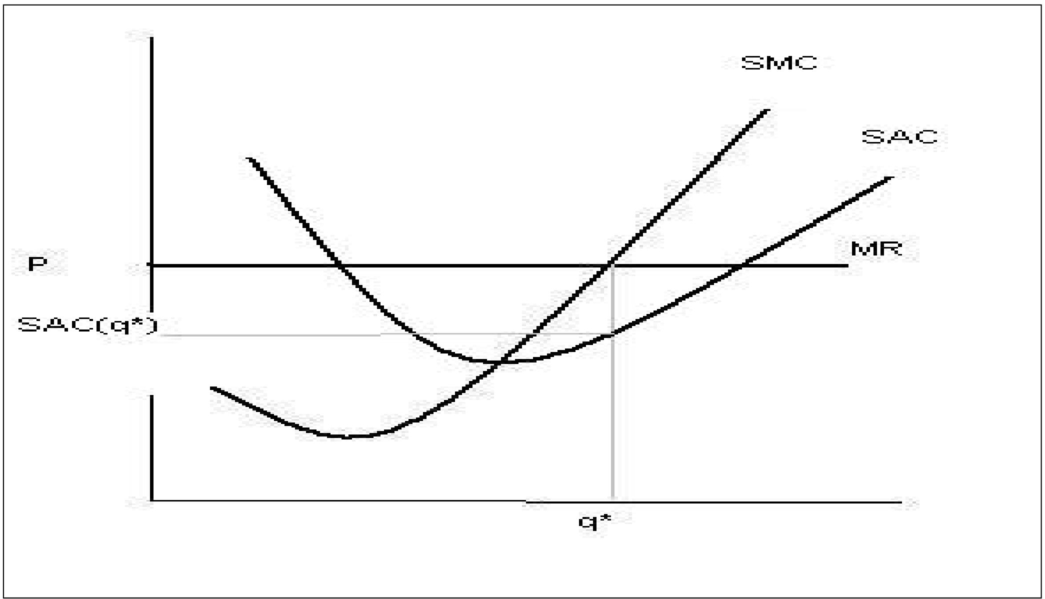


Figure 4: Profit Maximization of price maker



Recall from Principles that a firm will shut down if it can't afford to pay its variable costs. Shut

Figure 5: Profit of price taker



down if:

$$\begin{aligned} Rev &< SVC \\ pq &< SVC \\ p &< \frac{SVC}{q} \\ p &< SAVC \end{aligned}$$

This allows us to identify what the supply curve for a price-taking firm will look like. At any price below $SAVC$, $q^* = 0$. At a price $\geq SAVC$, the firm will supply where $MR = MC$. Therefore, its supply curve at these prices is shown by SMC.

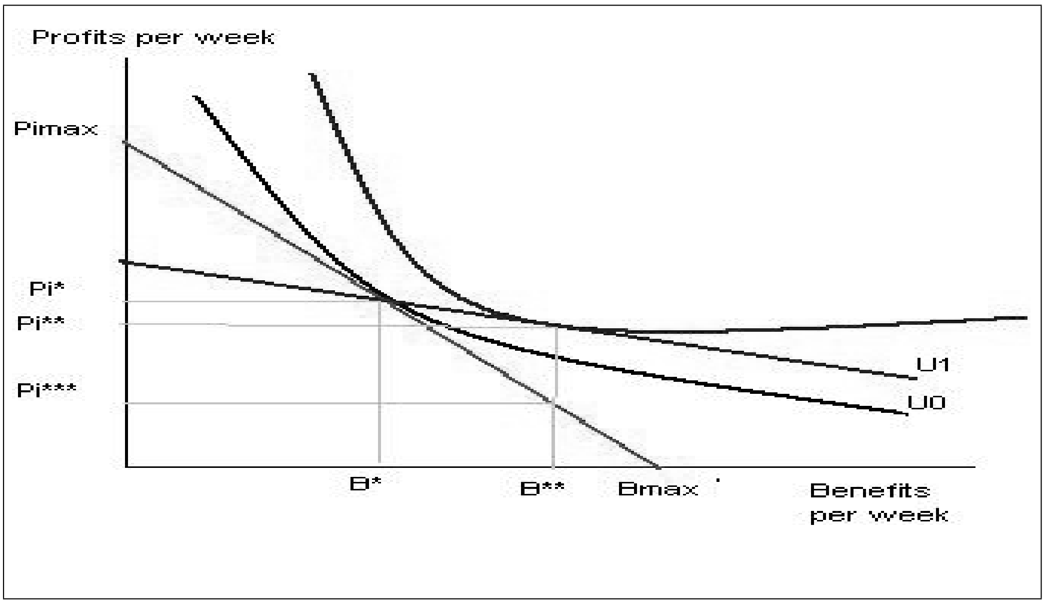
5 Principal Agent

So far, we have acted as if the owner of the firm was also the manager of the firm. Thus when the manager maximized profits for the firm, she also maximized profits for herself. In other words, it would be in the owner/manager's best interest to maximize the firm's profits.

In fact, it is often the case that the manager and the owner are different. It in fact may not be in the manager's best interest to maximize profits even though this is what the owner wants him to do. The principal-agent problem refers to the problem that the manager may not do what the owner wants him to do. Let's consider the principal-agent problem graphically.

Let's consider the preferences over the firm's profit and benefits (such as power, jets, and good dinners.) We can draw indifference curves over these two goods. First, suppose that the owner is the manager. Each \$1 spent on benefits is \$1 less of profits. So the budget constraint has a slope of -1. If there were no benefits, the interecept on the π axis would be $\pi_{\max}$. If there were no profits, the intercept on the benefit axis would be $B_{\max}$. The maximization point is at π^*, B^*.

Figure 6: Principal Agent Problem



Now suppose that the owner is different from the manager. Suppose that the manager owns 33% of the stock and that outside investors own 67% of the stock. From the manager's perspective, every additional \$1 spent on benefits only costs her \$0.33 in lost profits. Thus, from the manager's perspective, the budget constraint is $-\frac{1}{3}$. Thus, the manager would choose to operate at π^{**}, B^{**}. But in reality, each \$1 spent on benefits costs \$1 in lost profits. So when the manager chooses to consume B^{**} benefits, profits are just π^{***}. The firm's owners are hurt by the fact that the manager is operating differently than they would like her to. (In general, the less manager owns of the company, the greater the difference in actions by the manager and actions desired by the owners.) Owners try to deal with this problem by creating incentive-compatible contracts. This thinking can explain profit-sharing contracts and stock options.

5.1 Examples

The principal agent problem is a common one in economics. Other examples include: investment advisors (do they put their client's interest first) and mechanics (do they put your interests first when deciding whether repairs are needed.)

5.1.1 Franchising

Franchising can be seen as an attempt to solve the principal-agent problem. McDonald's are typically owned by a local businessperson who pays a fee to McDonalds. If McDonalds owned all its restaurants and hired managers to run them, managers would have little incentive to make the business run efficiently. Instead each restaurant is owned by an individual who has an incentive for it to run efficiently and who lives close enough to keep close tabs on the business. This increases the overall profit of McDonalds.

5.1.2 Doctors and Patients

We go to doctors because we don't have full information about health and medical treatments. We (principal) expect a doctor to act on our behalf (agent). But in fact the doctor might make different decisions for us than we would make for ourselves if we had full information. For example, the doctor likely does not consider (or even know) the cost of different treatments. She will simply be looking for the "best" treatment, regardless of cost. A similar problem is encountered when we think about professors ordering books for their students.