

# 1 Characteristics of Oligopoly

- Few firms
- Homogeneous goods
- Some pricing power
- Restricted entry

## 2 Cartel

A cartel is a set of firms that coordinate their output decisions. In essence, they act like a monopoly. They choose to produce the amount that will maximize total profits for the set of firms. The typical example of a cartel is OPEC.

There are a number of reasons why the cartel solution is usually not sustainable over a long period of time. First, it is illegal to form cartels in the US under the Sherman Act of 1890. Second, the cartel solution requires a great deal of information about costs and demand that the firms might not have or might not wish to share. Finally, cartels often fall apart because individual members have an incentive to cheat.

### 2.1 OPEC

OPEC frequently decides on a total amount of oil that should be produced. This amount is designed to maximize profits for OPEC. Individual members are given assigned amounts that they can produce. It is rational for one country to think that if it increased its output slightly above its designated level it would enjoy higher profits, given the high price of oil. The problem is that every country thinks in this way, causing total output to increase quite a bit, lowering the price. In addition, if one country thinks that another will increase its output and that the price will fall, it makes sense to try and produce more to make up for the lower price.

### 2.2 DeBeers

DeBeers is another famous example of a cartel. The company was founded in South Africa by Cecil Rhodes. Until 1900, DeBeers was essentially a monopoly as it controlled 99% of diamond. Now DeBeers owns approximately 15% of diamond mines. However, it controls approximately 80% of diamonds through the Central Selling Organization. This is the clearinghouse for almost all diamonds. Offer diamond merchants small boxes of assorted diamonds on "take it or leave it" basis. If merchant doesn't buy, have to wait until invited again. Because it then controls the amount of new diamonds that enter the market, it is able to maintain a high price for diamonds. If diamond mine owners attempt to go around DeBeers, it floods the market for diamonds so as to make it unprofitable to sell the diamonds.

### 3 Cournot Model (Simultaneous Quantity Setting)

The basic idea of the Cournot model is that two firms figure out how much to produce in order to maximize their profits. The model assumes that each firm decides how much to produce at the same time. Each firm takes into account that there is another firm producing and that this will affect the price they receive. However, neither firm takes into account that how much it produces will affect how much the other firm produces. The model assumes that each knows the demand in the market and the cost faced by all firms.

Total output in the market is the sum of the output of the two firms. Denote this as  $Q = q_1 + q_2$ . Assume that there is linear demand in the market:  $P = a - b(Q) = a - b(q_1 + q_2)$ . For the moment, keep things simple and assume that it is costless to produce the good for both firms.

Firm 1 thinks to itself about what firm 2's profit maximizing strategy will be. It knows that firm 2's problem is:

$$\begin{aligned} \underset{q_2}{Max} [a - b(q_1 + q_2)] q_2 \\ \underset{q_2}{Max} aq_2 - bq_1q_2 - bq_2^2 \\ a - bq_1 - 2bq_2 &= 0 \\ q_2 &= \frac{a - bq_1}{2b}. \end{aligned}$$

This represents firm 2's reaction function. It says how much firm 2 will produce when profit-maximizing given how much firm 1 produces.

Firm 2 is going through the same process and trying to figure out how much firm 1 will produce. We don't need to go through the math this time because both firms have the same cost (0), so each firm's profit-maximizing problem is symmetric. So firm 1's output will be

$$q_1 = \frac{a - bq_2}{2b}.$$

This is firm 1's reaction function. It says how much firm 1 will produce when profit-maximizing given how much firm 2 produces.

We can think about this problem graphically. The graph shows both firm's reaction functions. The Cournot equilibrium is at  $q_1^*, q_2^*$ . This point shows how much each will produce when profit-maximizing and knowing the profit-maximizing strategy for the other firm. To understand this equilibrium, consider point  $A$ , which is not an equilibrium. At this point, firm 1 thinks that if firm 2 produces  $q_{2A}$ , then it will produce  $q_{1A}$ . But firm 2 thinks that if firm 1 produces  $q_{1A}$  it will produce  $q'_{2A}$ . Firm 1 having gone through all this in his head knows then that he should not produce  $q_{1A}$  because the only time it would be profit maximizing for him to produce  $q_{1A}$  is if firm 2 was going to produce  $q_{2A}$  and he knows that firm 2 won't produce  $q_{2A}$  if he does  $q_{1A}$ . So  $A$  can't be an equilibrium.

$q_1^*, q_2^*$ , however, is an equilibrium. At this point, firm 1 knows that it is profit-maximizing for him to produce  $q_1^*$  only if firm 2 produces  $q_2^*$  but he knows that it is profit-maximizing for firm 2 to do this. Both of them will go through this thought process and determine that  $q_1^*, q_2^*$  is the best strategy. A Cournot equilibrium can be thought as the output levels were each firm makes the correct assumption about what the other firm will produce.

This graph shows us a way to solve the problem. We see that the equilibrium solution occurs where the two reaction functions are equal to each other. Therefore,

$$\begin{aligned} q_2 &= \frac{a - b\left(\frac{a - bq_2}{2b}\right)}{2b} \\ q_2 2b &= a - b\left(\frac{a - bq_2}{2b}\right) \\ q_2 2b &= a - \left(\frac{a - bq_2}{2}\right) \\ bq_2\left(2 - \frac{1}{2}\right) &= \frac{a}{2} \\ \frac{3}{2}bq_2 &= \frac{a}{2} \\ q_2 &= \frac{a}{3b} \end{aligned}$$

Substituting into a reaction function,

$$\begin{aligned} q_1 &= \frac{a - b\left(\frac{a}{3b}\right)}{2b} \\ &= \frac{a - \frac{a}{3}}{2b} \\ &= \frac{2a}{6b} \\ &= \frac{a}{3b}. \end{aligned}$$

So,  $(q_1^*, q_2^*) = \left(\frac{a}{3b}, \frac{a}{3b}\right)$ . Each should produce the same amount.

Actually, in this particular case we should have known they would produce the same amount and we could solve for the exact amount a lot easier. Recall, that the two firms are the same because they face the same costs (0). Therefore, we know that  $q_1 = q_2$ . Thus,

$$\begin{aligned} q_2 &= \frac{a - bq_2}{2b} \\ 2bq_2 &= a - bq_2 \\ 3bq_2 &= a \\ q_2 &= q_1 = \frac{a}{3b} \end{aligned}$$

**Example 1**  $P = 80 - Q; MC_1 = MC_2 = 10$ . Calculate the output produced by each and profit of each firm.

**Example 2** How do total profits in the above Cournot problem compare with total profits in the cartel situation?

**Example 3**  $P = 80 - Q; MC_1 = 10; MC_2 = 5$

## 4 Price Leadership (Competitive Fringe) Model

The idea of this type of model is that there are some markets where one firm is dominant. It tends to lead the way in setting the price and other smaller firms follow the dominant firm's lead. Examples include US Steel Corporation prior to WWII and IBM in the early computer market.

Lets first illustrate this model graphically. Assume there is one price-setting leader and a bunch of firms who take the price leader's price as the given price. Call this group of firms the competitive fringe.

$D_m$  is the market demand.  $S_{cf}$  is the market supply of the competitive fringe. At a price less than  $P_2$ ,  $S_{cf}$  tells us that the nothing would be supplied by the competitive fringe. This implies that everything must be supplied by the dominant firm. Thus at  $P_2$ , the demand for the market is the same as the demand faced by the dominant firm. At a price of  $P_1$ , the graph shows us that the demand in the market is equal to the amount supplied by the competitive fringe. Thus, at  $P_1$  the dominant firm is supplying nothing, meanings that its demand is 0 at this price. Given these two endpoints, we can infer that the demand for the dominant firm  $D_{df}$  connects these two points. For any price between  $P_1$  and  $P_2$ , we can therefore determine how much is produced by the dominant firm and how much is produced by the competitive fringe.

So now lets consider how our dominant firm acts. He is a price maker and sets his  $MR = MC$ . Given that, we observe that the dominant firm will produce  $Q_{df}$  and charge a price of  $P^*$ . The firms in the competitive fringe take this price as given. Looking at the interesection of  $P^*$  and  $S_{cf}$ , we then observe that at this price, the competitive fringe firms will produce  $Q_{cf}$ . We know that at  $P^*$ , the total amount demanded in the market is  $Q^*$ , where  $Q_{cf} + Q_{df} = Q^*$ .

We can also solve this problem mathematically. Lets start by thinking about the follower's problem. We know that these firms are price-takers and must charge the same price as the leader. Suppose  $Q = a - bP$ ,  $c_2(q_2) = \frac{q_2^2}{2}$ ,  $c_1(q_1) = cq_1$ . The follower's problem is:

$$\begin{aligned} \underset{q_2}{Max} \quad & pq_2 - \frac{q_2^2}{2} \\ p - q_2 &= 0 \\ q_2 &= p. \end{aligned}$$

The leader knows that the amount it sells plus the amount sold by the competitive fringe equals the total amount sold in the market and that how much

the competitive fringe produces is determined by the price it sets. The leader knows that:

$$\begin{aligned} q_1 &= Q - q_2, \\ &= a - bp - q_2 \\ &= a - bp - p \\ &= a - p(1 + b) \end{aligned}$$

So the leader's profit maximization problem is:

$$\begin{aligned} &Max\ q_1p - cq_1 \\ &Max\ p[a - p(1 + b)] - c[a - p(1 + b)] \\ &Max\ (p - c)[a - p(1 + b)] \\ a - p(1 + b) - (1 + b)(p - c) &= 0 \\ a - (1 + b)(2p - c) &= 0 \\ p &= \left[ \frac{a}{1 + b} + c \right] \frac{1}{2} \\ &= \frac{a + c(1 + b)}{2(1 + b)} \end{aligned}$$

So,

$$\begin{aligned} q_1 &= a - (1 + b) \left[ \frac{a + c(1 + b)}{2(1 + b)} \right] \\ &= a - \frac{a + c(1 + b)}{2} \\ &= \frac{a - c(1 + b)}{2}. \end{aligned}$$

and

$$\begin{aligned} q_2 &= Q - q_1 \\ &= Q - \frac{a - c(1 + b)}{2} \end{aligned}$$

**Example 4** Suppose that you are on the board of directors of a firm that is the price leader in the industry. It lets all the other firms sell all they want at the prevailing market price. In other words, the other firms act as perfect competitors. Your firm sets the price that all the other firms accept. The demand curve for your industry's product is  $P = 300 - Y$ . The total amount supplied by the other firms is equal to  $Y_r = 49P$ ; this is the supply curve for the competitive fringe. Your firm's  $MC = 2.96Y_b$  where  $Y_b$  is the output of your firm. What output would maximize your profit? What price should you charge? How much would the industry as a whole produce at this price?

## 5 Quantity Leadership (Stackelburg Model)

One limitation with the Cournot model is that while it models the fact that price will be affected by the total output of both firms, it doesn't take into account that how much one firm chooses to produce may affect how much the other chooses to produce. Like the competitive fringe model, the Stackelburg model assumes that there is a leader in the market who makes an initial output decisions. This firm needs to determine how much to produce in order to maximize profits, taking into account that the price at which it can sell will be determined by total output in the market. In other words, the leader firm needs to figure out how the other firm will react to its initial output decision.

Lets start out by considering the same original problem as for the Cournot Model. This will allow us to compare the two results. Total output in the market is the sum of the output of the two firms. Denote this as  $Q = q_1 + q_2$ . Assume that there is linear demand in the market:  $P = a - b(Q) = a - b(q_1 + q_2)$ . For the moment, keep things simple and assume that it is costless to produce the good for both firms.

The Leader firm thinks about the Follower firm's profit maximization problem.

$$\begin{aligned} \underset{q_2}{Max} [a - b(q_1 + q_2)] q_2 \\ a - bq_1 - 2bq_2 &= 0 \\ q_2 &= \frac{a - bq_1}{2b} \end{aligned}$$

So far, this looks like the Cournot problem. We have calculated firm 2's reaction function, which tells how the follower's output decision will vary dependent on the leader's output decision.

Having determined how the follower's output will vary given how much he, the leader, produces, the leader starts considering his own profit maximization problem.

$$\underset{q_1}{Max} [a - b(q_1 + q_2)] q_1 \text{ s.t. } q_2 = \frac{a - bq_1}{2b}$$

Here we see the crucial difference. The leader is deciding how much he should produce so as to maximize his profits, taking into account how the follower will react. Rewriting,

$$\underset{q_1}{Max} \left[ a - b \left( q_1 + \frac{a - bq_1}{2b} \right) \right] q_1$$

In other words, the leader realizes that his own output decision determines how much the follower will produce. Knowing how the follower will react, the leader's

output decision determines everything.

$$\begin{aligned} \text{Max}_{q_1} \quad & aq_1 - b \left( q_1^2 + \left( \frac{a - bq_1}{2b} \right) q_1 \right) \\ \text{Max}_{q_1} \quad & \frac{aq_1}{2} - \frac{bq_1^2}{2} \\ \frac{a}{2} - bq_1 &= 0 \\ q_1^* &= \frac{a}{2b} \end{aligned}$$

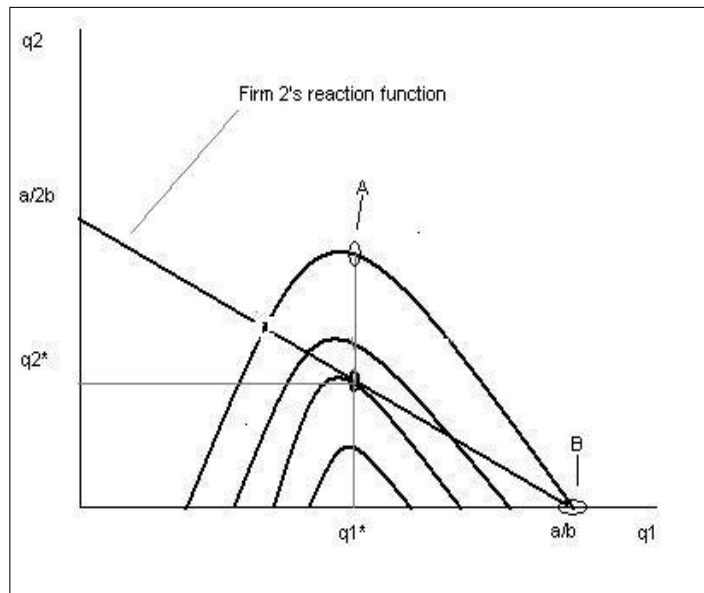
So we have determined how much firm 1 should produce. It is easy then to determine how much firm 2 will produce.

$$\begin{aligned} q_2 &= \frac{a - b \left( \frac{a}{2b} \right)}{2b} \\ q_2^* &= \frac{a}{4b} \end{aligned}$$

Lets now interpret these results graphically. In the Stackelburg model, we only have one reaction function, that of the follower, firm 2. On the graph, I have drawn some isoprofit lines. An isoprofit lines shows all the output combinations of firm 1 and 2 that give a constant level of profit to firm 2. Along one isoprofit line, the profit level is the same for firm 2. Isoprofit lines that are closer to the  $q_1$  axis are associated with a higher profit than those further from the axis. To understand this, consider point  $A$  and the point of  $q_1^*, q_2^*$ . Both involve the same amount of  $q_1, q_1^*$ . Firm 2 is producing more at point  $A$  than at  $q_1^*, q_2^*$ . Since firm 1 is producing the same amount in either case, this means that total output is higher at point  $A$ , which means the price is lower, and therefore, profit is lower for firm 1. Having interpreted the isoprofit curves, lets consider the Stackelburg equilibrium. Firm 1 knows that for any amount it produces, how much firm 2 will produce. This is shown by firm 2's reaction function. Therefore, firm 1 knows that firm 2 must produce somewhere along its reaction function. Firm 1 wants to get the highest profit possible. In other words, firm 1 wants to be on the lowest possible isoprofit curve it can attain. The highest possible profit occurs on the second lowest isoprofit curve in this graph. The lowest isoprofit curve is not attainable. Why not choose point  $B$ ? From the above argument,  $B$  is associated with a lower profit, so you wouldn't choose this. How can this be you ask? It is obvious from the graph that at point  $B$ , firm 1 is producing more and firm 2 is producing nothing. How could this not lead to a higher profit for firm 1 than the point  $q_1^*, q_2^*$ ? The intuitive explanation for this is the same as the reason why the monopolist doesn't produce as much as people would buy; the more you produce, the lower the price you get. At point  $B$ , the amount produced by firm 1 must be greater than total output  $q_1^* + q_2^*$ . resulting in "too low" a price. Even though firm 1 is producing more, he is past the quantity that maximizes profits. So the Stackelburg equilibrium is  $q_1^* + q_2^*$

**Example 5**  $P = 80 - Q; MC_1 = MC_2 = 10$ . For the Stackelburg model, calculate the output produced by each and profit of each firm.

Figure 1: Stackelberg Model



**Example 6**  $P = 80 - Q; MC_1 = 10; MC_2 = 5$ . For the Stackelberg model, calculate the output produced by each and profit of each firm.

### 5.1 Comparison to Cournot Equilibrium

Consider the following graph to compare the Cournot and Stackelberg equilibriums. I have added firm 1's reaction function to illustrate the Cournot equilibrium. Graphically, we see that in this same cost case, Firm 1 produces more and Firm 2 produces less in the Stackelberg model than in the Cournot model. This is the intuitive result we would expect. In the Cournot model, the two firms determine their output levels at the same time. In the Stackelberg model, firm 1 essentially has more power because it determines its output first and firm 2 responds.

Figure 2: Stackelberg vs Cournot Model

