Market Demand and Elasticity

March 31, 2004

Review Principles material

1 Constructing Market Demand

$x_i^1(p_1, p_2, m_i)$ = consumer $i's$ demand for good 1 given market prices p_1 and p_2 and $i's$ income m

Suppose n consumers

Market demand=sum of all individuals demand for a particular good. Therefore,

$$X^1(p_1, p_2, m_1, ...m_n) = \sum_{i=1}^{n} X_i^1(p_1, p_2, m_i)$$

meaning that the market demand for good 1 given market prices p_1 and p_2 and all individual incomes = the sum of individual demand for good 1 for all n consumers.

Example 1 *Tom, Dick, and Harry are the entire market for scrod. Tom's demand curve is*

$$Q_T = \left\{ \begin{array}{c} 100 - 2P, \ P \leq 50 \\ 0, \ P > 50 \end{array} \right\}.$$

Dick's demand curve is

$$Q_D = \left\{ \begin{array}{c} 160 - 4P, \ P \leq 40 \\ 0, \ P > 40 \end{array} \right\}.$$

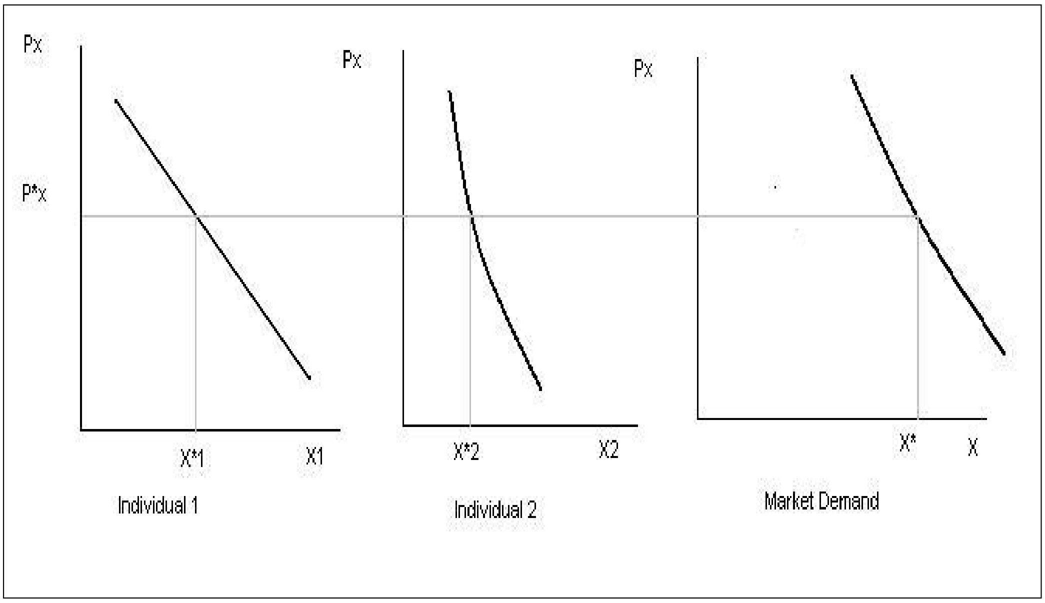
Harry's demand curve is

$$Q_H = \left\{ \begin{array}{c} 150 - 5P, \ P \leq 30 \\ 0, \ P > 30 \end{array} \right\}.$$

What is the market demand?

$$Q = \left\{ \begin{array}{c} 0, \ P > 50 \\ 100 - 2P, \ 40 < P \leq 50 \\ 260 - 6P, \ 30 < P \leq 40 \\ 410 - 11P, \ P \leq 30 \end{array} \right\}$$

Figure 1: Adding individual demand curves to get market demand



1.1 Shifts in Market Demand

Since market demand is just the sum of individual demands, market demand will shift from anything that shifts individual demand. So what sort of things shift individual demand? Recall,

$$Q_x^d = f\left(P_x, P_y, m; preferences\right)$$

Demand is a relationship between the price of the good and the quantity of the good. Therefore, individual demand (and hence market demand) shifts from

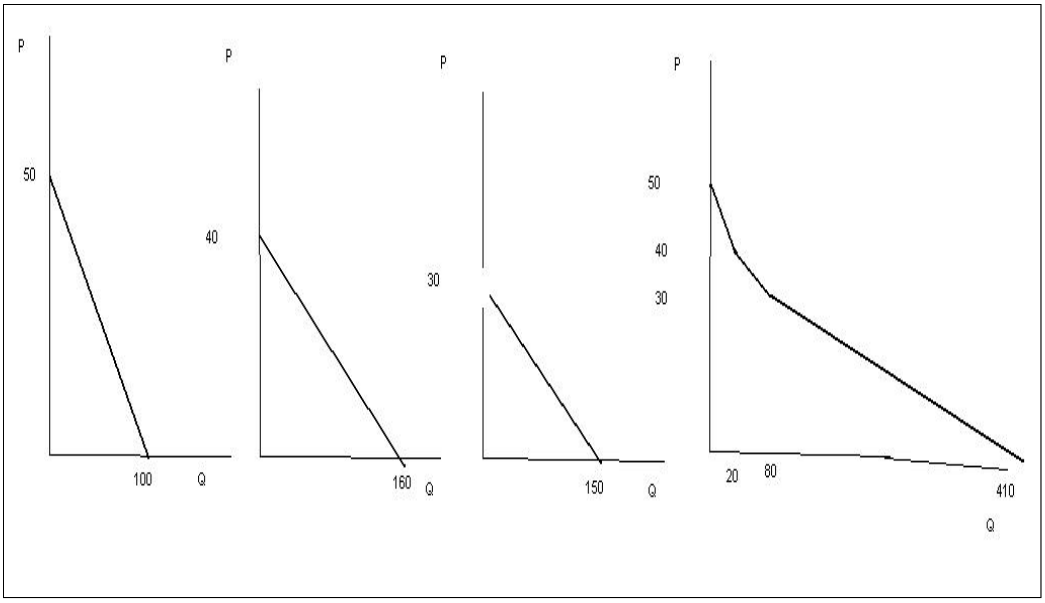
- change in income
- change in price of related good
- change in preferences or any of the other things we were holding constant.

In addition, since market demand is the sum of individual demands, market demand will shift as the number of consumers changes.

2 Elasticity

Measure of responsiveness - How much does one variable change in response to another variable?
Elastic - very responsive
Inelastic - not very responsive

Figure 2: Example of adding demand curves



2.1 Why not use slope?

It tells us how much one variable changes when another variable changes.

Problem: the slope measure depends on the units used.

Example 2 *Suppose we want to know whether steak or eggs are more responsive to a price change. Steak usually quoted as a per pound price; eggs are quoted per dozen.*

$$\begin{aligned} \textit{slope of steak demand} &= \frac{\textit{Change in quantity (pounds)}}{\textit{change in price}} \\ \textit{slope of egg demand} &= \frac{\textit{Change in quantity (dozen)}}{\textit{change in price}} \end{aligned}$$

When you do your calculation, how do you compare them? One tells you change in pounds per dollar while the other tells you change in dozen per dollar.

Example 3 *Suppose you know that when the price of doodles was \$2, 80 were purchased. When the price was \$1, 100 were purchased.*

$$\textit{slope}=\frac{-20\ \textit{units}}{1\ \textit{dollar}} = -20$$

Suppose you converted prices into cents (200 cents and 100 cents respectively). Now,

$$\textit{slope} = \frac{-20\ \textit{units}}{100\ \textit{cents}} = -0.2$$

This is the same exact good and prices, but we get different measures of responsiveness when we use slope, depending on the way we quote the price.

So, slope is not a good measure of responsiveness. We want a unit-free measure. We use elasticity.

2.2 Elasticity of demand (ε^d)

Recall from your principles class that since ε^d tells us how much quantity demanded changes given a change in price, we can write this as:

$$\varepsilon^d = \frac{\%\Delta Q^d}{\%\Delta P}$$

Recall that if a good is very responsive, Q^d will change a lot relative to P. Therefore, $\varepsilon^d > 1$. Having quanity change a lot relative to price is the meaning of elastic; therefore, $\varepsilon^d > 1$ is the definition of an elastic good.

Note: By the law of demand, we know (for a non-Giffen good) that price and quantity demanded move in opposite directions. Therefore, ε^d will always be a negative number. Oftentimes, the absolute value of this measure is reported. That is what I will do. The book reports the negative sign.

By a similar logic, an inelastic good will be one where $\varepsilon^d < 1$. A unit elastic good is one where $\varepsilon^d = 1$.

Exercise 4 *Using the above formula for elasticity, explain intuitively why $\varepsilon^d < 1$ is an inelastic good.*

Recall that in Principles, we typically calculated ε^d using the midpoint formula. Let's go through this procedure to see how the derivative measure we'll use for this class is essentially the same thing.

The midpoint formula is just a way of defining how you will calculate a percentage change; you use the average as the base.

$$\begin{aligned}\varepsilon^d &= \frac{\%\Delta Q^d}{\%\Delta P} \\ &= \frac{\frac{Q_1^d - Q_0^d}{Q_A^d}}{\frac{P_1 - P_0}{P_A}} \\ &= \left(\frac{Q_1^d - Q_0^d}{Q_A^d}\right)\left(\frac{P_A}{P_1 - P_0}\right)\end{aligned}$$

where Q_A denotes the average quantity, etc. Rewriting this a little bit, we get

$$\begin{aligned}\varepsilon^d &= \left(\frac{Q_1^d - Q_0^d}{P_1 - P_0}\right)\left(\frac{P_A}{Q_A^d}\right) \\ &= \left(\frac{\Delta Q^d}{\Delta P}\right)\left(\frac{P_A}{Q_A^d}\right)\end{aligned}$$

Now, if we want to deal with very small changes, we can think in calculus terms. Rewriting this, we get

$$\varepsilon^d = \frac{dQ^d}{dP}\frac{P}{Q}.$$

In other words, we take the derivative of the demand function with respect to price and multiply it by the respective price and quantity.

2.2.1 Relationship between ε^d and Revenue

Intuitively

Definition 5 *Revenue is the amount of money collected from selling a good*

$$Revenue = P * q$$

Example 6 *Suppose you decrease the price a little and demand for the good is elastic. What happens to revenue?*

$$R = P * Q$$

Since the good is elastic, quantity demanded will increase a lot when you decrease the price. So P decreases a little and Q increases a lot. The effect on Q (output effect) is greater than the effect on P (price effect) so revenue will increase. Intuitively, when you decrease the price a little, people are very responsive to price change and buy a lot. So even though you are selling at lower price, quantity increases so much that revenue increases.

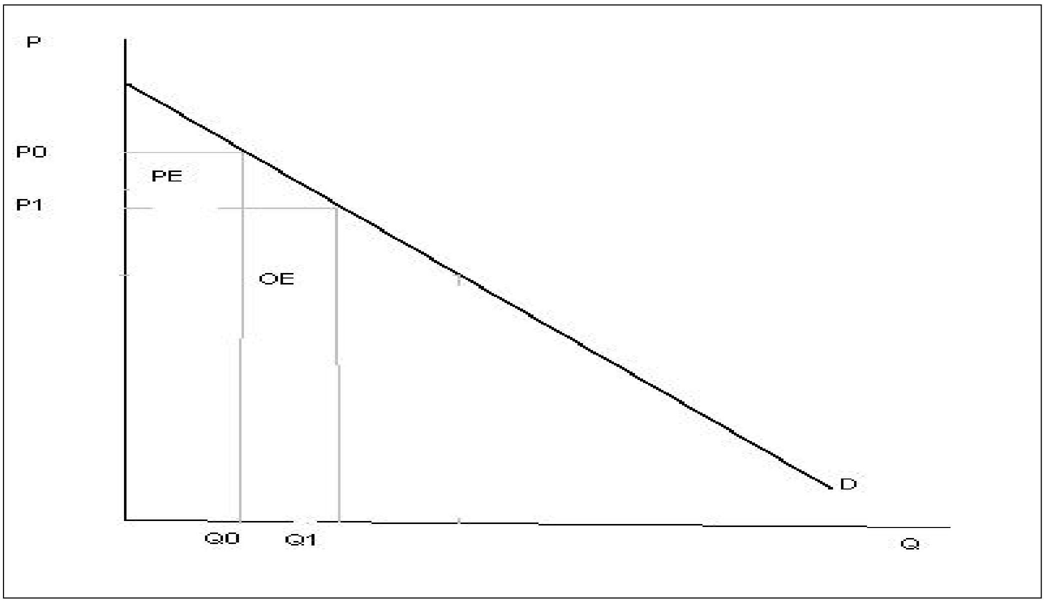
This example illustrates that there are two effects of a price change: the price effect and the output effect. The price effect tells us the effect on revenue from changing the price, imagining that the quantity didn't change. The output effect says what is the effect on revenue from just changing quantity. Obviously, the two effects have opposite effects on revenue. Whichever effect is stronger will determine the effect on revenue. We can also see these two effects graphically. Again, suppose the demand for a good is elastic. The price falls from P_0 to P_1, causing quantity to increase from Q_0 to Q_1. Asking what happens to revenue (does it increase or decrease) is the same as asking is the new revenue ($P_1 * Q_1$) larger or smaller than the old revenue ($P_0 * Q_0$)? The way I've drawn it, the output effect is larger than the price effect; therefore, revenue increases.

On your own, go through the following exercises and fill out the table.

Exercise 7 *Suppose you increase the price a little and demand for the good is elastic. What happens to revenue?*

Exercise 8 *Suppose you increase the price a little and demand for the good is inelastic. What happens to revenue?*

Figure 3: Output Effect versus the Price Effect



Exercise 9 *Suppose you decrease the price a little and demand for the good is inelastic. What happens to revenue?*

Elasticity	**Price Change**	**Revenue Change**
Elastic	Price increases	
Elastic	Price decreases	
Inelastic	Price increases	
Inelastic	Price decreases	

Mathematically To understand the above mathematically, we will make use of the product rule.

Definition 10 *Product Rule: if* $f(x) = g(x) * h(x)$ *then* $f'(x) = g'(x) * h(x) + h'(x)\, g(x)$. *In words, this is "Derivative of the first times the second plus the derivative of the second times the first".*

Example 11

$$\begin{aligned} f(x) &= \left(x^2+3x+1\right)(5x+2) \\ f'(x) &= \left(x^2+3x+1\right)'(5x+2)+\left(x^2+3x+1\right)(5x+2)' \\ &= (2x+3)(5x+2)+\left(x^2+3x+1\right)(5) \\ &= 15x^2+34x+11 \end{aligned}$$

Observe that we can write our revenue function as

$$R(p) = p * q(p).$$

This says that revenue is a function of price. Revenue is price time quantity like we said earlier. But now we note that quantity is affected by price, denoted as $q(p)$.

Note that above we were asking what was the effect of a change in price on revenue. This is same as saying what is the derivative of the revenue function with respect to price. By the product rule,

$$\begin{aligned} R'(p) &= 1 * q(p) + p * \frac{dq}{dp} \\ &= q(p) + p * \frac{dq}{dp} \end{aligned}$$

We want to explore the mathematical relationship between revenue and elasticity, so lets rewrite the above in terms of elasticity. Now that we've taken the derivative, lets just use the notation, $q = q(p)$

$$\begin{aligned} R'(p) &= q + p * \frac{dq}{dp} \\ &= q\left[1 + \frac{p}{q}\frac{dq}{dp}\right] \\ &= q\left[1 - \left|\varepsilon^d\right|\right] \end{aligned}$$

If $\varepsilon^d < 1$ then $R'(p) > 0$. So, if demand is inelastic then an increase in price cause revenue to increase.

Application: Agriculture Demand for agricultural products tends to be quite inelastic. What does this tell us about the effect of bad weather effects (such as hurricanes, droughts, etc) or technological improvements? Bad weather causes a decrease in supply. We know that this will cause an increase in price. Because demand is inelastic, equilibrium quantity doesn't go down that much. What will happen to revenue? From above logic, we know that revenue will in fact increase.

2.2.2 ε^d along a demand curve

In most cases, elasticity changes along a demand curve.

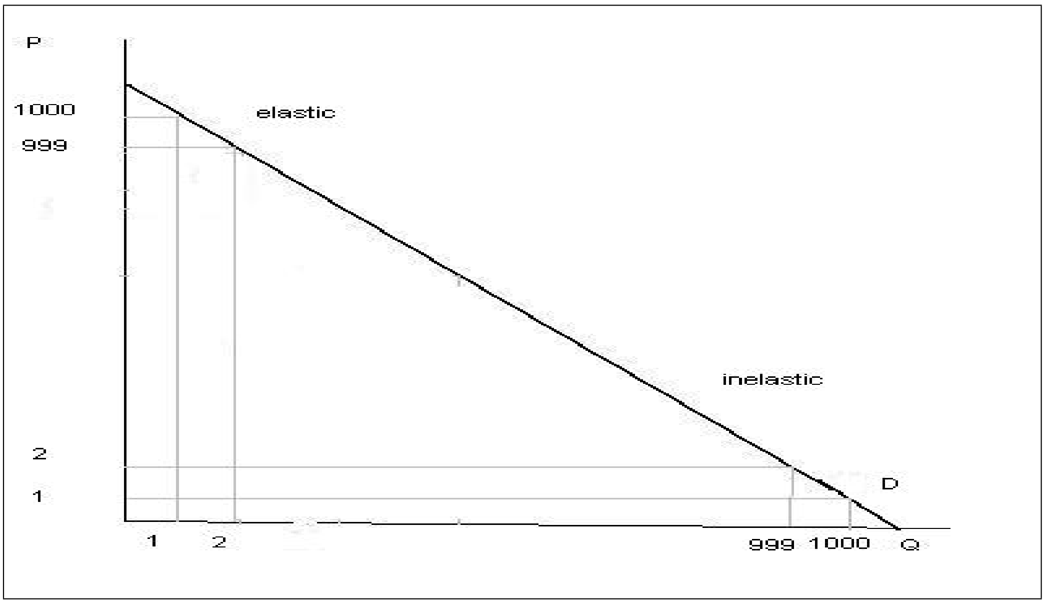
Linear Demand curve This is most obvious in the case of a linear demand curve. Lets first see this intutively using a graph and then mathematically. In the graph, consider a price increase from \$1 to \$2 and the subsequent decrease from 1000 to 999. Using the intuitive elasticity formula,

$$\begin{aligned} \varepsilon^d &= \frac{\%\Delta Q^d}{\%\Delta P} \\ &= \frac{\text{small}}{\text{big}} \\ &< 1. \end{aligned}$$

Similarly, if consider a price increase from \$999 to \$1000 and the subsequent decrease from 2 to 1. Then,

$$\begin{aligned} \varepsilon^d &= \frac{\%\Delta Q^d}{\%\Delta P} \\ &= \frac{\text{big}}{\text{small}} \\ &> 1. \end{aligned}$$

Figure 4: Elasticity Along a Linear Demand Curve



Lets see why this is happening mathematically. Consider a linear demand curve,

$$Q = 1001 - P.$$

Applying our elasticity formula,

$$\begin{aligned}\varepsilon^d &= \frac{dQ^d}{dP}\frac{P}{Q} \\ &= -1 * \frac{P}{Q} \\ &= -\frac{P}{Q}\end{aligned}$$

In this example, we see that the elasticity will depend on the ratio of price to quantity. If $P > Q$ then demand at that point will be elastic; if $P < Q$ then demand at that point will be inelastic.

Example 12 *Let $Q = 100 - 2P$. Calculate elasticity at $P = 40$ and $P = 25$ and $P = 10$. What happens to revenue as you change price?*

Constant Elasticity Demand Curve There are certain (non-linear) demand curves, which have constant elasticity at all points along the curve. Consider the case of

$$Q = \frac{1200}{P}.$$

Then,

$$\begin{aligned}
\varepsilon^d &= \frac{dQ^d}{dP}\frac{P}{Q} \\
&= -\frac{1200}{P^2}\frac{P}{Q} \\
&= -\frac{1200}{P}\frac{1}{Q} \\
&= -\frac{1200}{P}\frac{1}{\frac{1200}{P}} \\
&= -1
\end{aligned}$$

Regardless of the price or quantity, you will always have unit elasticity.

Example 13 $X = \alpha P^\beta$

$$\begin{aligned}
\varepsilon^d &= \frac{dX}{dP}\frac{P}{X} \\
&= \alpha\beta P^{\beta-1}\frac{P}{X} \\
&= \frac{\alpha\beta P^\beta}{\alpha P^\beta} \\
&= \beta
\end{aligned}$$

So the elasticity of this demand function is β at all values of X.

Typically, we write the log form of this equation:

$$\ln X = \ln \alpha + \beta \ln P$$

Note that

$$\frac{d \ln X}{d \ln P} = \frac{dX}{dP}\frac{P}{X} = \varepsilon^d$$

So for this case,

$$\frac{d \ln X}{d \ln P} = \beta$$

3 Other Types of elasticities

3.1 Income Elasticity (ε^m)

Says how quantity demanded changes when income changes (through shift in demand)

$$\begin{aligned}
\varepsilon^m &= \frac{dq}{dm}\frac{m}{q} \\
\varepsilon^m &> 0 \quad \text{normal} \\
\varepsilon^m &< 0 \quad \text{inferior}
\end{aligned}$$

3.2 Cross-Price Elasticity (ε^{xy})

Says how quantity demanded changes when the price of another good changes (through shift in demand)

$$\begin{aligned} \varepsilon^{xy} &= \frac{dq_x}{dp_y}\frac{p_y}{q_x} \\ \varepsilon^{xy} &> 0 \text{ substitutes} \\ \varepsilon^{xy} &< 0 \text{ complements} \end{aligned}$$

3.3 Price elasticity of supply (ε^s)

Says how quantity supplied changes when price changes.

$$\begin{aligned} \varepsilon^{s} &= \frac{dq^s}{dp}\frac{p}{q^s} \\ \varepsilon^{s} &> 1 \quad \text{elastic} \\ \varepsilon^{s} &< 1 \quad \text{inelastic} \end{aligned}$$

Table 1: Example elasticities

Item	ε^d	ε^m
Food	−0.21%	0.28%
Medical Services	−0.18%	0.22%
Rental Housing	−0.18%	1.00%
Owner-Occupied Housing	−1.20%	1.20%
Electricity	−1.14%	0.61%
Automobiles	−1.20%	3.00%
Beer	−0.26%	0.38%
Wine	−0.88%	0.97%
Marijuana	−1.50%	0.00%
Cigarettes	−0.35%	0.50%
Abortions	−0.81%	0.79%
Transatlantic air travel	−1.30%	1.40%
Imports	−0.58%	2.73%
Money	−0.40%	1.00%

4 Empirical Estimation of Demand

Typically we assume either market demand has a constant slope or market demand has a constant elasticity.

4.1 Linear

How can we estimate demand using observable data? The table below shows the quantity of raspberries sold in a market each year. The price and quantity data are shown in the graph below. If

Table 2: Example demand data

Year	Quantity(Q)	Price(P)	Income(M)
1988	4	\$24	\$10
1989	7	\$20	\$10
1990	8	\$17	\$10
1991	13	\$17	\$17
1992	16	\$10	\$17
1993	15	\$15	\$17
1994	19	\$12	\$20
1995	20	\$9	\$20
1996	22	\$5	\$20

we believe that price alone determines demand, it would be reasonable to draw a line which fits the points (D), using an equation of the form

$$Q = \alpha + \beta P.$$

It is only reasonable to think that this curve fits the data if no important factors other than price affect demand. Note in the table, however, that we have included the variable income, which shows three different levels. This suggests that demand has shifted over time. The demand curves, $d1, d2, d3$, give a more likely description of demand. These demand curves can be described by the form

$$Q = \alpha + \beta P + \gamma m.$$

Note that this equation allows the demand curve to shift in a parallel fashion as income changes.

4.2 Constant Elasticity

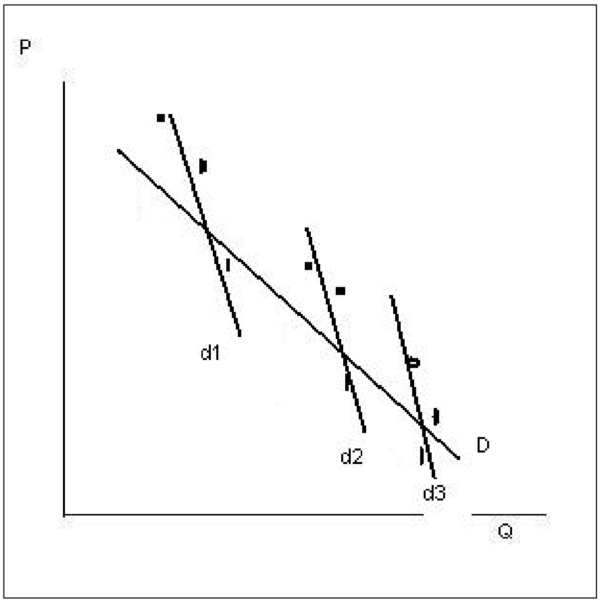
The same idea holds for estimating a demand function with constant elasticity. All that changes is the form of the equation. Again, we have observations of Q^d at different prices. Controlling for other variables,

$$\ln X = \ln \alpha + \beta_{p_x} \ln P_x + \beta_m \ln m + \beta_{p_y} \ln P_y + \beta_{p_z} \ln P_z + ...$$

4.3 Application: Coffee (1963-1977)

Demand tends to be stable. Supply shifts a lot. Therefore, we have a lot of observed points. This study observed that for regular coffee, elasticity of demand was nearly the same at high and low

Figure 5: Estimation of a Linear Demand Curve



prices. Therefore, constant elasticity of demand was appropriate. Based on collected data, the econometric estimation of demand was:

$$\ln C = -0.16 \ln P_c + 0.51 \ln m + 0.15 \ln P_t - 0.009 \ln T + \alpha$$

where

$$\begin{aligned} C &= \text{quantity of coffee} \\ P_c &= \text{price of coffee} \\ m &= \text{income} \\ P_t &= \text{price of tea} \\ T &= \text{time} \end{aligned}$$

These estimates show that $\varepsilon^d = -0.16$. Demand for coffee is quite inelastic. A 10% increase in the price of coffee will lead to a 1.6% decrease in quanity demanded.

$\varepsilon^m = 0.51$. This is a positive. As income increases, demand will increase. This is a normal good. A 10% increase in income will lead to a 5.1% increase in demand.

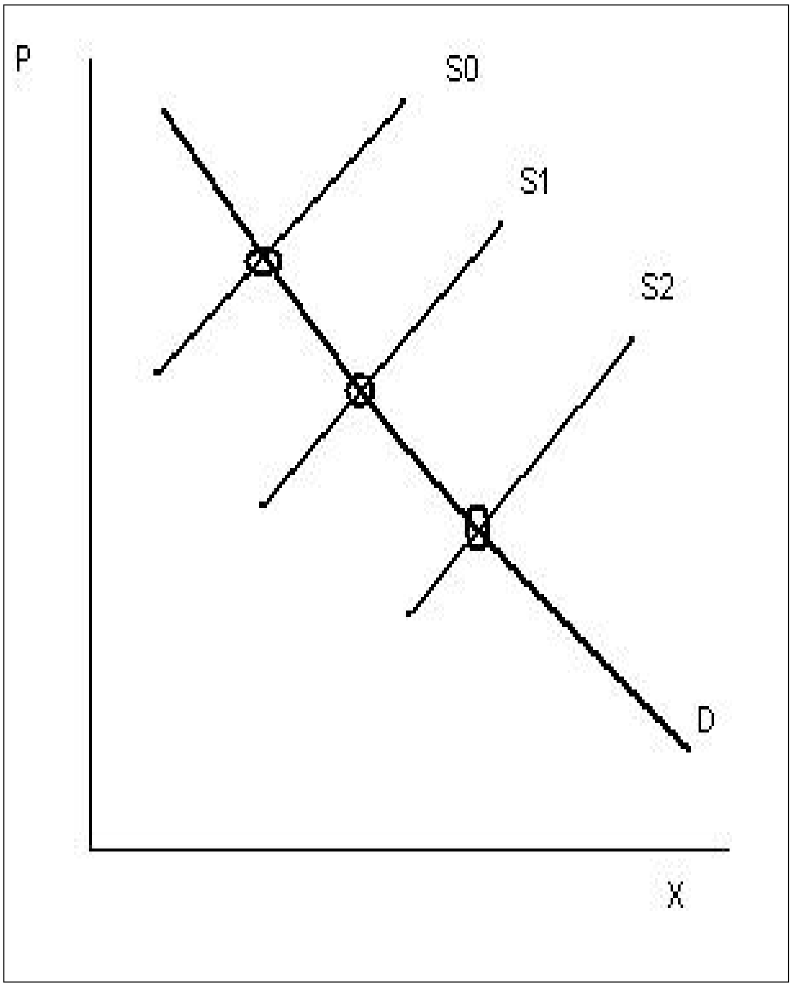
$\varepsilon^{ct} = 0.15$. Coffee and tea are substitutes. A 10% increase in the price of tea will lead to a 1.5% decrease in the demand for coffee.

$\varepsilon^T = -0.009$. Over time, people are drinking less coffee.

4.4 Application: Pork

$$Q_p = 171 - 20P_p + 20P_b + 3P_c + 2m$$

Figure 6: Multiple Supply Shifts Give Observed Demand Points



where

$$
\begin{aligned}
Q_p &= \text{quantity of pork (in millions of kg)} \\
P_p &= \text{price of pork} \\
P_b &= \text{price of beef} \\
P_c &= \text{price of chicken} \\
m &= \text{income (in thousands)}
\end{aligned}
$$

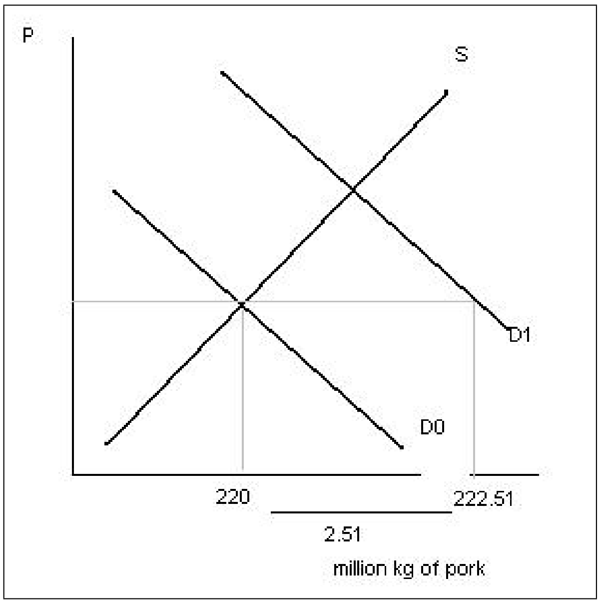
Suppose income is 12.5, the quanity of pork purchased is 220, the price of beef is \$4/kg, and the price of chicken is \$2/kg.

$$\begin{aligned}\varepsilon^m &= 2\frac{m}{q} \\ &= 2\frac{12.5}{220} = 0.114\end{aligned}$$

Pork is a normal (but not superior) good. A 10% in income will cause a 1.14% increase in pork purchased. This means that the demand curve will shift right by

$$0.0114 * 220 = 2.51 \text{ million kg}$$

Figure 7: Pork Market



$$\begin{aligned}\varepsilon^{pb} &= 20\frac{p_b}{q_p} \\ &= 20\frac{4}{220} = 0.364 > 0\end{aligned}$$

Pork and beef are substitutes. A 10% increase in the price of beef will cause a 3.64% increase in the quantity demanded of pork.

$$\begin{aligned}\varepsilon^{pc} &= 3\frac{p_c}{q_p} \\ &= 3\frac{2}{220} = 0.027 > 0\end{aligned}$$

Pork and chicken are substitutes. A 10% increase in the price of chicken will cause a 0.27% increase in the quantity demanded of pork. Pork is a stronger substitute for beef than it is for chicken.