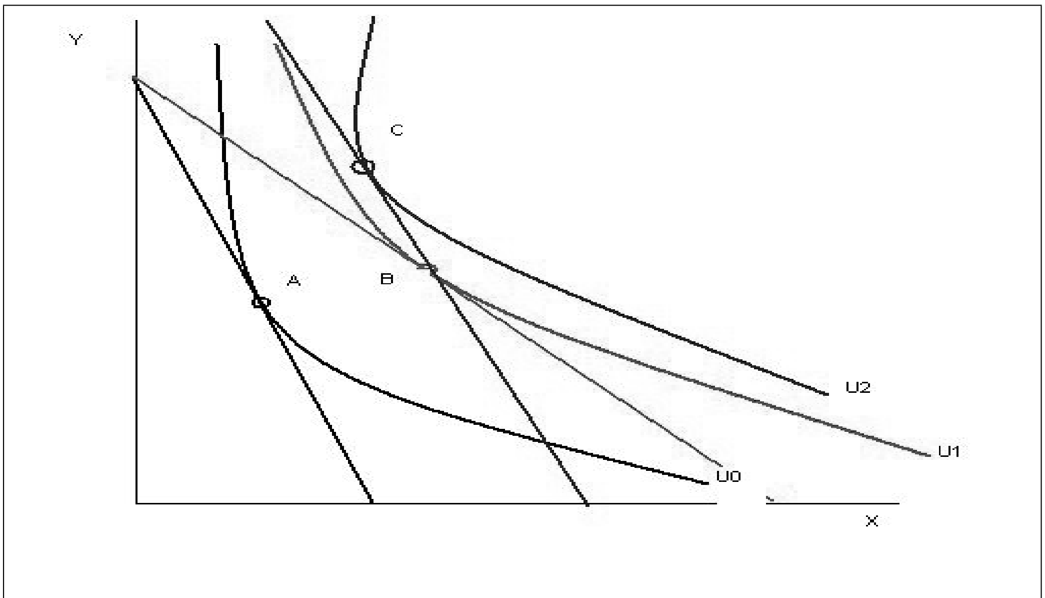
Individual Demand continued

February 16, 2004

1 Application: In-Kind Aid

Figure 1: Provision of In-Kind Aid versus Income Assistance



Aid programs in the US consist of income assistance (TANF) and in-kind aid (food stamps, subsidized housing). The graph below shows that for the same amount of direct government assistance per person, the assistee gains more welfare from income asssistance than from in-kind assistance. A represents the point that the indvidual would consume if no assistance was provided. Consider in-kind assistance that essentially lowers the price of good x. This causes the budget constraint to pivot out. Given this assistance, the individual would consume bundle B. Now suppose that instead of providing in-kind assistance, the government provided enough income assistance such that the individual could again afford B. This income assistance is equivalent to an income increase where the prices don't change. Given this new budget constraint, we see that the individual would choose to consume bundle C instead of bundle B. This tells us that providing the a certain amount of money through income assistance provides a greater increase in utility level than spending the same amount of income on in-kind assistance.

This raises the obvious question of why we provide in-kind aid rather than cash assistance. Possible answers include that the government only wants to assistance to be spent on certain types of goods, potential connections to farm policies, and/or a reflection about government beliefs about individual preferences.

2 Deriving a Demand Curve

Recall what a demand curve is. It shows the relationship between price and quantity demanded; it shows us for all prices, what quantity an individual would demand. We know that our utility maximization model also shows us the bundle that an individual will choose when they are trying to maximize their utility subject to a budget constraint. Since a bundle consists of two goods, this suggests that there will be a tight connection between the two graphs. Since the demand curve shows the relationship between price and quantity demanded, we need to examine a utilty maximization model where the price is changing.

Consider the graph below. The top graph shows the different bundles that will be chosen as the price of X changes. The bottom graph shows a subset of this data. It shows the amount of X that will be chosen as the price of X changes. At a price of P_x', X' maximizes utility. We can plot this point in (P, X) space. As the price of X decreases to $P_x^{"}$, the amount of X that maximizes utility is X". We can also plot this point in (P, X) space, which gives us a second point on the demand curve (since nothing else has changed - the ceteris paribus assumption). Etc.

Voila! We now know why demand curves are drawn as negatively sloped. This is the result of the fact that for all normal goods the substitition effect and income effect move in the same direction (opposite of price) and for all known inferior goods the substituion effect is greater than the income effect. Since we have shown with indifference curve analysis that this causes a decrease in price to result in an increase in the amount consumed of a good, then our derived demand curves must follow the law of demand.

2.1 Shift of income curves

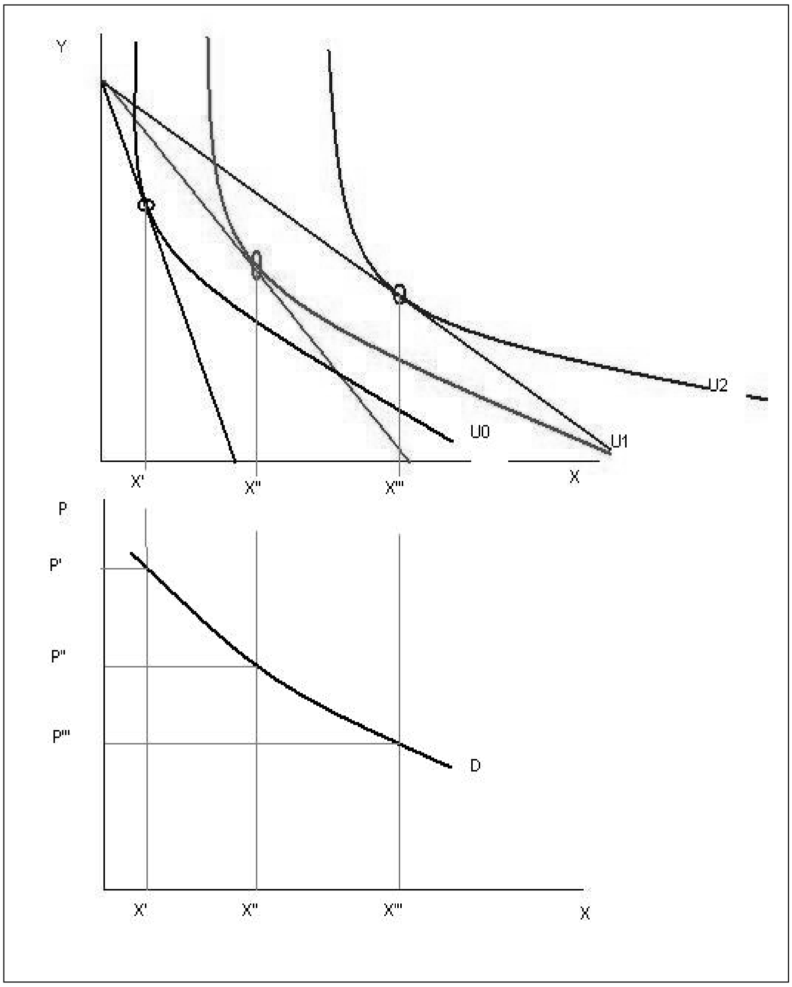
As we learned in principles, the demand curves shifts when any of the things we've been holding constant (income, price of other goods, preferences, etc) have changed.

3 Measuring Consumer Welfare Changes

Motivation: In enacting a policy, goverment would like to figure out whether individuals are better or worse off under the policy. To do cost-benefit analysis, need to attach dollar amount to this. Essentially, want to figure out utility change and monetize it. 3 ways to do this:

- compensating variation
- equivalent variation

Figure 2: Derivation of a Demand Curve



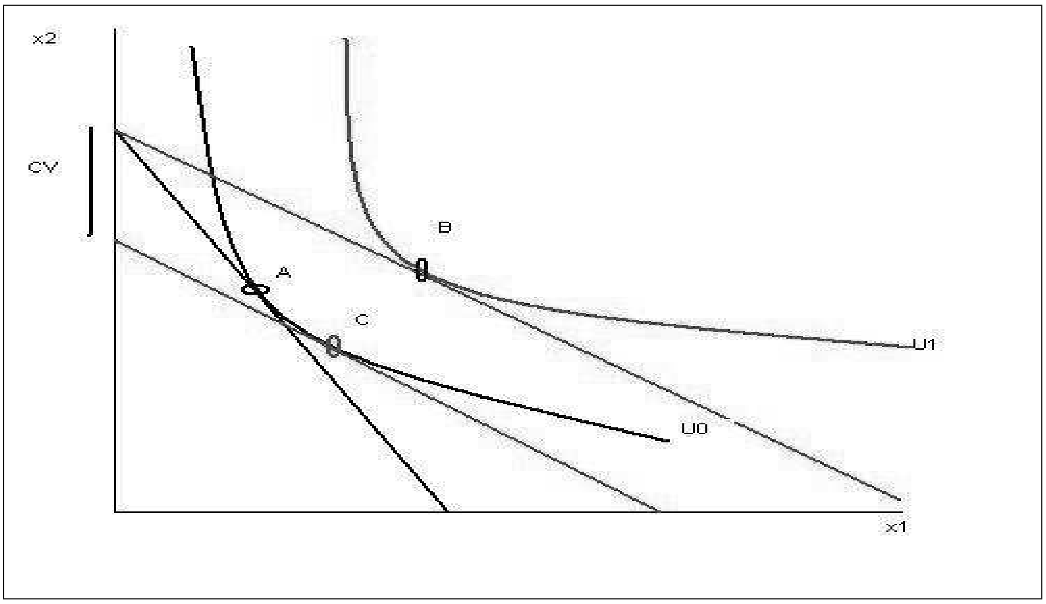
- consumer surplus

3.1 Compensating Variation

Question: How much money would have to give/take away from an individual after a price change to make her as well off as before the price change? In other words, how much money would we have to give/take away after the price change so that the individiual can achieve their original utility level. This amount is the compensating variation. It is called compensating variation because it is the amount of money required to compensate (after the fact) for a price change.

In the graph below, the individual originally consumes A. Suppose the price of good 1 decreases. The budget constraint pivots out and the new selected bundle would be B. Because the price of good 1 went down, the individual is better off. We want to know how much money we would have to take away from this person, at the new prices, so that she could achieve the same utility level as before the price change. We could shift this new budget constraint in until it is tangent to the original utility, U_0. The difference between the two vertical intercepts is CV. Why? For simplicity, we assume that $p_2 = 1$. Because the vertical intercepts are $\frac{m}{p_2}$ and $\frac{m'}{p_2}$, when $p_2 = 1$, the difference between these two intercepts is the change in income.

Figure 3: Compensating Variation for a decrease in p_1



Exercise 1 *Draw and explain CV for the case of an increase in* p_1

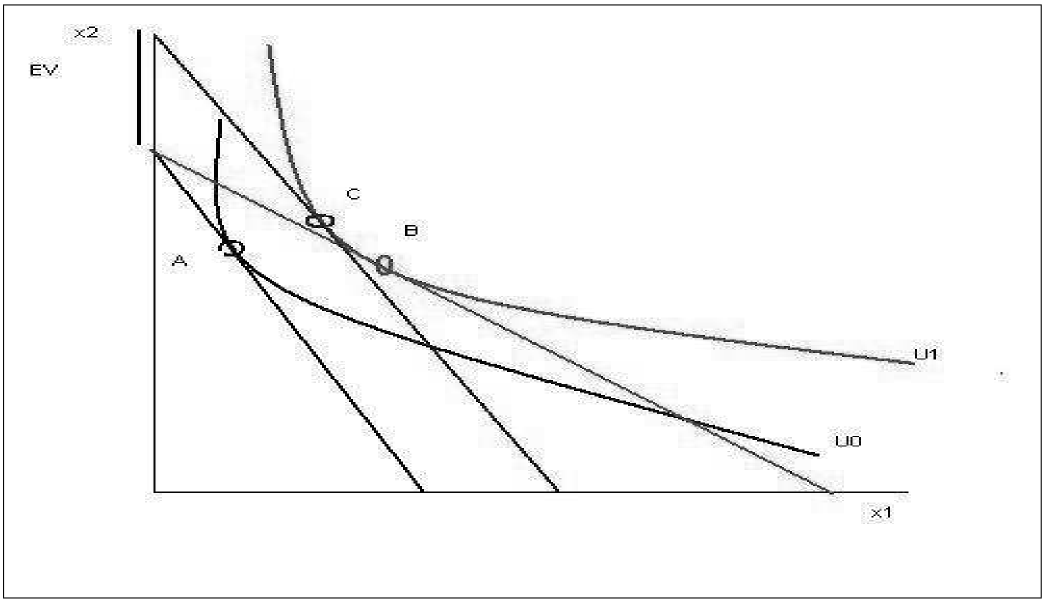
Exercise 2 *Draw and explain CV for the case of an increase in* p_2

Exercise 3 *Draw and explain CV for the case of an decrease in* p_2

3.2 Equivalent Variation

Question: how much money would we have to give/take away from an individual before the price change to make him as well off as before the price change? In other words, how much money would we have to take away from an individual before the price change to get him to the same utility he would be after the price change? This amount of money is the equivalent variation (EV). Again, it puts utility changes in dollar terms.

Figure 4: Equivalent Variation for a decrease in p_1



In the graph above, the individual originally consumes A. Suppose the price of good 1 decreases. The budget constraint pivots out and the new selected bundle would be B. Because the price of good 1 went down, the individual is better off. We want to know how much money we would have to give this person, at the old prices, so that she could achieve the same utility level as with the price change. We could shift this original budget constraint out until it is tangent to the new utility, U_1. The difference between the two vertical intercepts is EV. Why? For simplicity, we assume that $p_2 = 1$. Because the vertical intercepts are $\frac{m}{p_2}$ and $\frac{m'}{p_2}$, when $p_2 = 1$, the difference between these two intercepts is the change in income.

EV measures the maximum amount (in this case) that the individual would be willing to have taken away in order to be indifferent to the price increase.

Exercise 4 *Draw and explain EV for the case of an increase in* p_1

Exercise 5 *Draw and explain EV for the case of an increase in* p_2

Exercise 6 *Draw and explain EV for the case of an decrease in* p_2

3.3 Consumer Surplus

In certain cases, one can calculate CV and EV. However, one get an approximate measure of a welfare change by calculating consumer surplus. It is not an exact measures, but under many circumstances it is a good estimate.

Recall, downward sloping demand curve shows declining marginal willingness to pay; the more you consume, the less you're willing to pay for the next unit.

$$CS = \text{willingness to pay-amount paid}$$

CS represents a bonus to the consumer.

Figure 5: Consumer Surplus

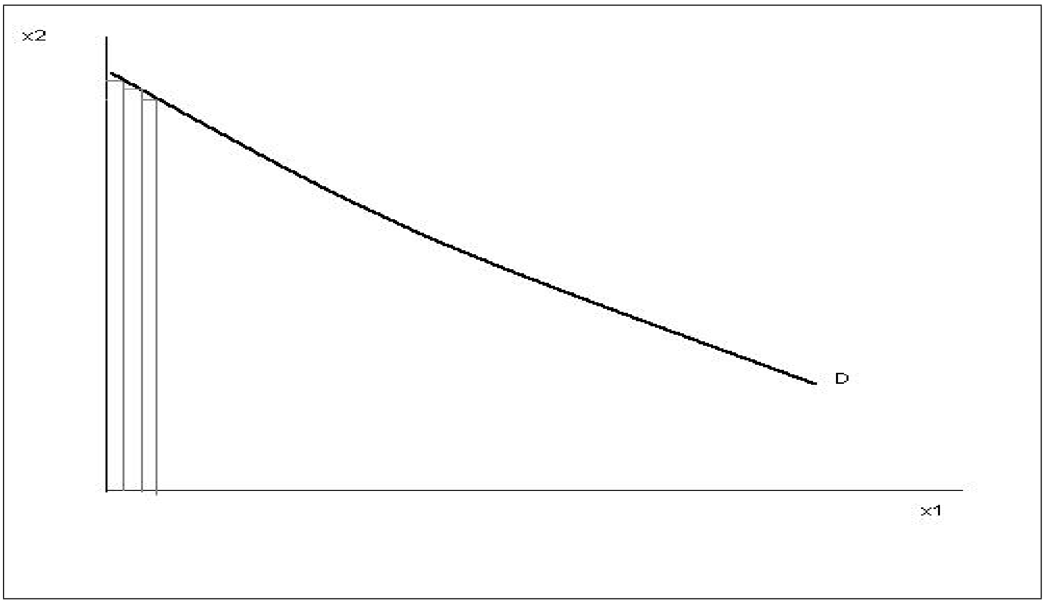
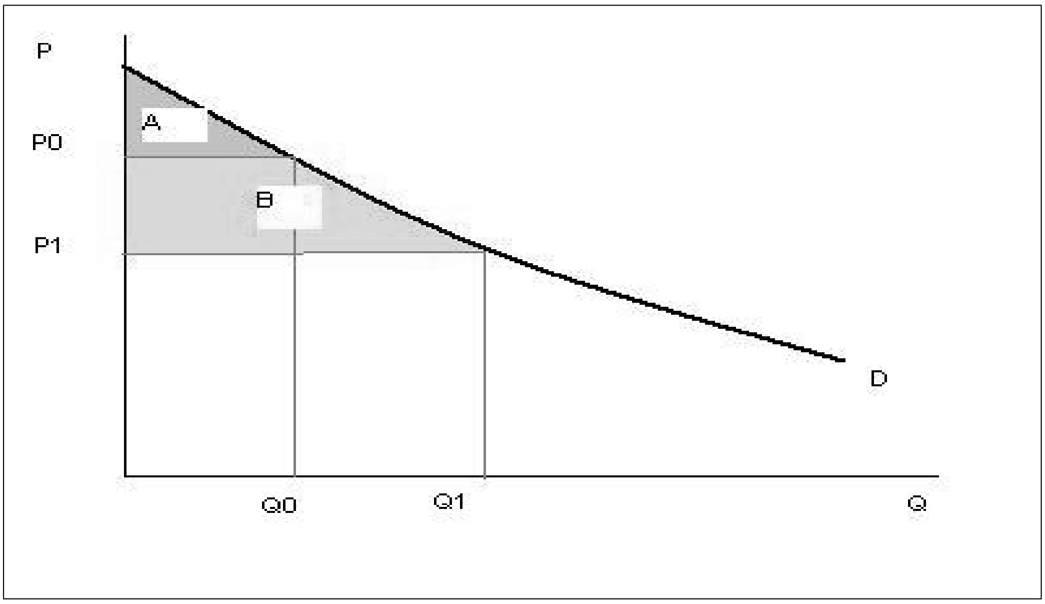


Figure 6: Consumer Surplus



Typically, we're interested in the change in consumer surplus. In the graph below, if the price falls from P_0 to P_1 then CS increases by area B. If the demand curve is linear, you can calculate this by adding up the area of the rectangle and triangle. If the demand curve is non-linear, then you can calculate it using an integral.

Exercise 7 *Suppose* $Q^d = 12 - P$ *and* $P^* = 8$. *What is consumer surplus?*

Exercise 8 *If* $P^* = 10$, *what is the change in consumer surplus?*

3.3.1 Consumer surplus through integrals

As noted above, one can also calculate the change in consumer surplus through the use of an integral. The basic idea of an integral is that you are undoing a derivative. When you calculate an integral, you are given a derivative and you want to figure out what the the original function was, before the derivative was taken. We will look only at the very simplest of integrals. An integral just calculates the area under a certain function.

Example 9 *add math notation*

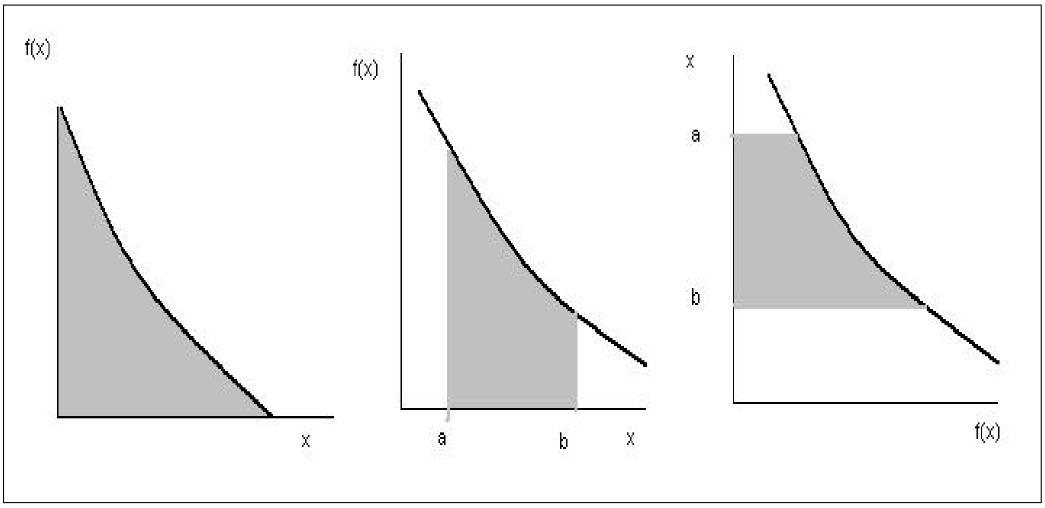
Example 10 $\int x \; dx = \frac{x^2}{2}$

Example 11 $\int x^2 \; dx = \frac{x^3}{3}$

Example 12 $\int 2x \; dx = x^2$

Example 13 $\int_b^a x^2 \; dx = \frac{x^3}{3}|_b^a = \frac{1}{3}(a^3 - b^3)$

Figure 7: Graphical Interpretation of Integrals



With the a, b notation, we are just calculating the area under a function between two endpoints, a and b.

Example 14 $\int_1^2 3x \; dx = \frac{3}{2}x^2|_1^2 = \frac{3}{2}\left(2^2 - 1^2\right) = \frac{3}{2}(3) = \frac{9}{2}$

Example 15 $\int_b^a \left(x^2 + 2x\right) \; dx = \left(\frac{x^3}{3} + \frac{x^2}{2}\right)|_b^a = \left(\frac{a^3}{3} + \frac{a^2}{2}\right) - \left(\frac{b^3}{3} + \frac{b^2}{2}\right)$

Example 16 *Consider the demand function, $x(p) = 20 - 2p$. The price changes from \$4 to \$6. Calculate the change in consumer surplus.*

$$\begin{aligned}
\int_4^6 (20 - 2p)\, dp &= \left(20p - p^2\right)|_4^6 \\
&= p(20 - p)|_4^6 \\
&= 6(14) - 4(16) \\
&= 4[21 - 16] \\
&= 20
\end{aligned}$$

Since this is a price increase, consumer surplus is negative. Consumer surplus has decreased by \$20. Note that this is a linear function, you could also calculate the change in consumer surplus in this case by calculating areas. In this case it is the area of a rectangle,

$$(6 - 4)\, 8 = 16,$$

plus the area of a triangle,

$$\frac{1}{2}(6 - 4)(12 - 8) = 4.$$

Adding together, we get that consumer surplus has decreased by \$20.

Example 17 *Consider the demand function, $x(p) = \frac{10}{p}$. The price changes from \$2 to \$3. Calculate the change in consumer surplus.*

$$\begin{aligned}
\int_2^3 \frac{10}{p} dp &= 10 \ln p|_2^3 \\
&= 10\left[\ln 3 - \ln 2\right] \\
&= 10 \ln \frac{3}{2}
\end{aligned}$$

Figure 8: CS change for price change from $4 to $6

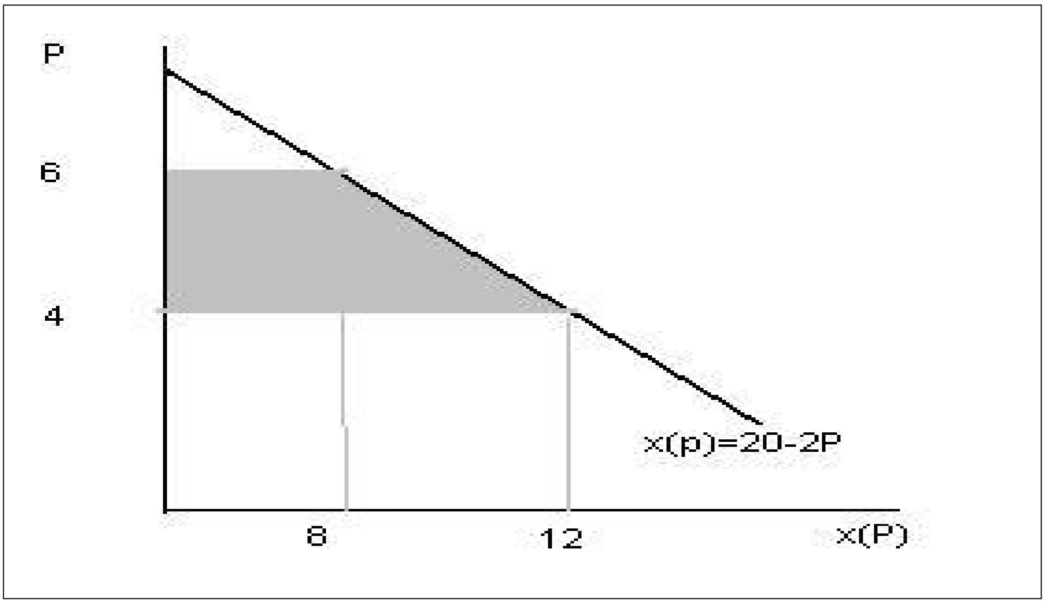


Figure 9: CS change for price change from $2 to $3