

1 Individual Demand Curves

Goal: Derive individual demand curves using choice model that we just derived

1.1 Demand functions

Definition 1 Define a demand function as

$$Q_x^d = f(P_x, P_y, m; \text{preferences})$$

The quantity demanded of good x is a function of the price of x , the price of another related good y , and income, given preferences. We assume those preferences are constant.

1.1.1 Homogeneity

Definition 2 We say a function $f(x)$ is homogeneous of degree r if

$$f(tx) = t^r f(x)$$

where t is some constant factor. Homogeneity tells us what happens to a function as all variables are changed by a proportion t . For example, if $t = 2$ we are asking what happens to the function if we double input x .

Example 3 $f(x) = x^2$

$$\begin{aligned} f(tx) &= (tx)^2 \\ &= t^2 x^2 \\ &= t^2 f(x) \end{aligned}$$

Therefore, this function is homogeneous of degree 2. If you double x , the function will increase by a factor of 4.

What is the homogeneity of a demand function? In other words, what happens to your demand if all variables double. Let's figure this out intuitively. Consider your budget constraint

$$\begin{aligned} p_1 x_1 + p_2 x_2 &= m \\ x_2 &= \frac{m - p_1 x_1}{p_2} \end{aligned}$$

What happens to the budget constraint if income and both prices double?

$$\begin{aligned} x_2 &= \frac{2m - 2p_1 x_1}{2p_2} \\ &= \frac{m - p_1 x_1}{p_2} \end{aligned}$$

In other words, the budget constraint doesn't change if you double all variables. You would get the same result if you changed all variables by a constant factor t . This makes intuitive sense: If income doubles but so does price, you couldn't afford to buy any more than before. Therefore, we say that demand is homogeneous of degree 0 :

$$\begin{aligned} f(tp_x, tp_y, tm) &= t^0 f(p_x, p_y, m) \\ &= f(p_x, p_y, m) \end{aligned}$$

1.1.2 Mathematical approach

Recall consumer problem:

$$\underset{x_1, x_2}{Max} U(x_1, x_2) \text{ s.t. } p_1 x_1 + p_2 x_2 = m$$

Previously, we showed that for

$$U(x_1, x_2) = \alpha \ln x_1 + (1 - \alpha) \ln x_2$$

we got the results

$$\begin{aligned} x_1 &= \alpha \frac{m}{p_1} \\ x_2 &= \frac{m}{p_2} (1 - \alpha) \end{aligned}$$

These are demand functions! They tell us how demand changes with m, p_1, p_2 . (Note: α is a some unspecified constant). Interestingly, for this utility function, we got the result that the demand for each good is a function of only income and the price of that good and not the price of another good.

1.2 Changes in Income

1.2.1 Both Goods are Normal

Recall, a normal good is one where you buy more as your income increases. This is shown in the graph below.

As your income increases, the budget constraint shifts out. We can see that each time, your consumption of x_1 and x_2 are increasing. Therefore, each must be normal goods.

1.2.2 One Good is Normal, One is Inferior

Recall, an inferior good is one where you buy less as your income increases. In the graph below, we see that as income increases, consumption of one good will increase but the other will decrease. Which one is inferior?

Figure 1: Normal Goods

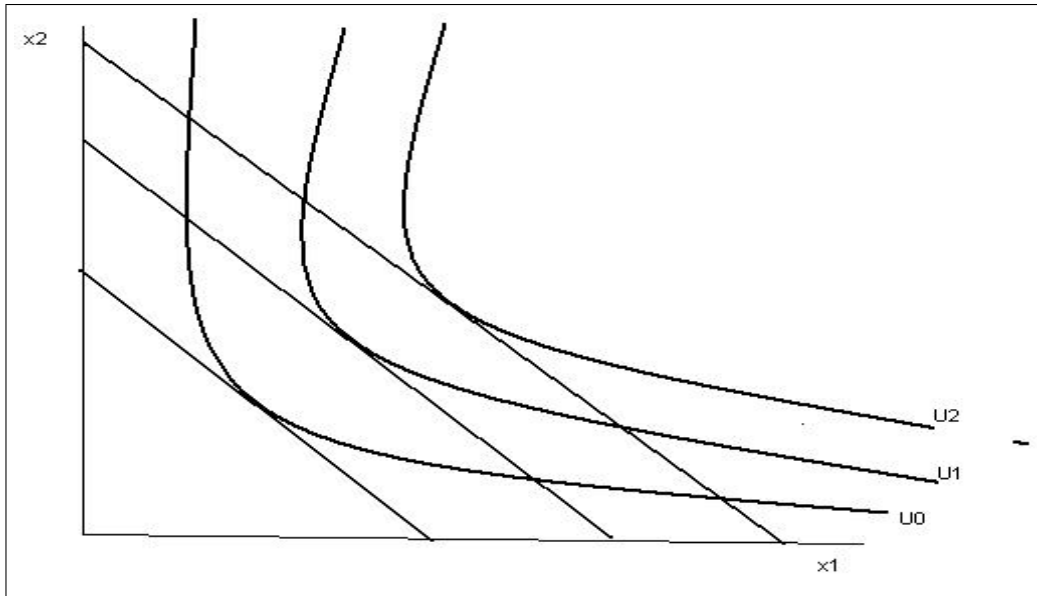
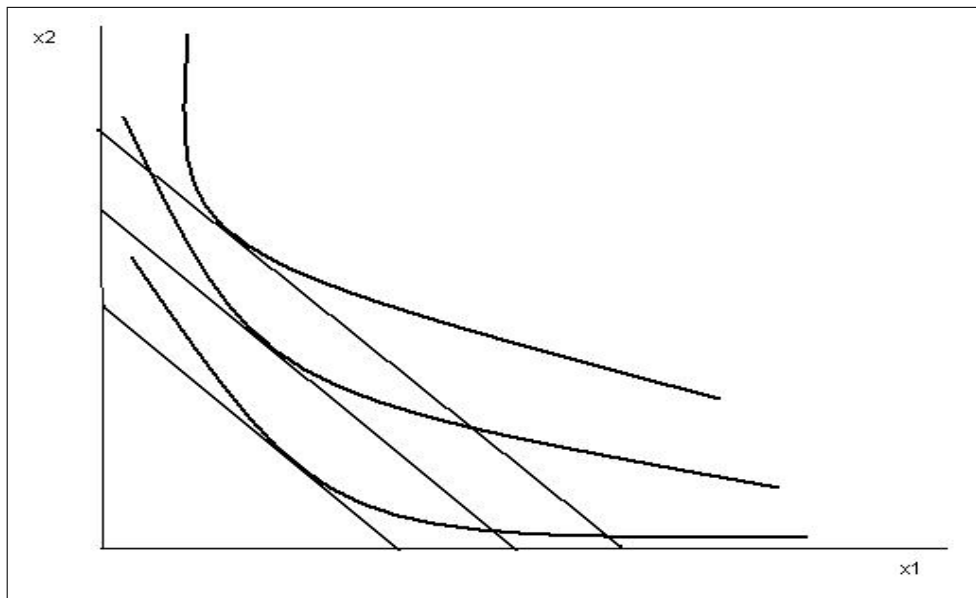


Figure 2: One Good Is Inferior



1.2.3 Application

One application of this type of analysis is to examine how spending on goods changes at different income levels. Are there certain goods that individuals would spend more on as their income increases

Table 1: Percentage of Total Expenditures by U.S. Consumers on Various Items, 2000; BLS

Item	\$15 – 20	\$40 – 50	\$70+
Food	15.4%	14.7%	11.4%
Clothing	4.8%	4.7%	5.3%
Housing	32.9%	30.4%	30.2%
Other items	46.9%	50.2%	53.1%

and others that they would spend less on? Consider the table below.

This table shows that as income increases, you spend a smaller share on food and housing and a larger share on clothing and other items.

1.2.4 Mathematically

We can also look at the derivatives of the demand functions to see what happens as income changes.

Exercise 4 *What happens to x_1 as income increases?*

$$\frac{\partial x_1}{\partial m} = \frac{1 + \alpha}{p_1} > 0$$

As income increases (and everything else is held constant), the quantity demanded of good 1 (x_1) increases (since $\alpha > 0$).

1.3 Price Changes

A price change has two effects on choices:

1. Substitution effect: As the price of one good becomes relatively more expensive you substitute towards the other good. This will change your consumption of x_1 and x_2 .
2. Income effect: As the price of a good increases, it means you have less income overall to spend. This will also change your consumption of x_1 and x_2 .

In graphical terms, this is akin to saying that a change in the price affects both the intercept and slope of the budget constraint.

We'll start out with normal goods.

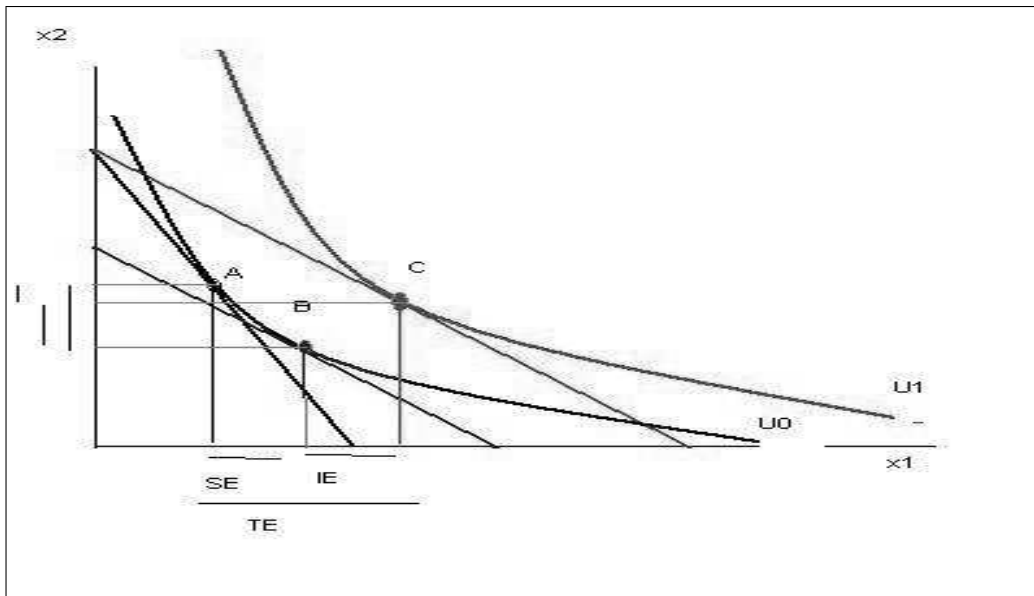
1.3.1 Normal goods

Decrease in price of good x_1 Lets work through the problem first intuitively. A decrease in the price of x_1 has two effects.

1. Substitution effect: You substitute some from x_2 to x_1 : x_1 increases, x_2 decreases
2. Income effect: A decrease in p_1 is equivalent to an income increase. If both goods are normal, then both x_1 and x_2 increase.

The end effect will be that consumption of x_1 unambiguously increases. What happens to x_2 is ambiguous; it depends on the magnitude of each effect.

Figure 3: Decrease in p_1 when x_1 is normal



Consider the graph above. At the original price, A is the optimal bundle. If there is a price decrease, we know the new optimal bundle will be C . We can break up the move from A to C into the substitution and income effects. First, consider the substitution effect. Suppose there was a change in price level but your income was the same such that you could still attain the original indifference curve. In this case, your new optimal bundle will be B . The move from A to B is the substitution effect. The decrease in p_1 also has an income effect. Because the decrease in p_1 is equivalent to an income increase, the budget constraint shifts out and the new optimal bundle is C . The move from B to C occurs simply because of the income change so the move from B to C is the income effect. The Total Effect is the combination of both the income and substitution effects. Graphically, we can see that we obtained the result that we expected from above. Both the income and substitution effects caused consumption of x_1 to increase. The way we have drawn the graph, we have shown x_2 decreasing slightly, meaning that for x_2 , the substitution effect was stronger than

the income effect. The graph below shows a situation where the income effect is stronger than the substitution good for x_2 .

For this situation must x_1 always increase?

Increase in price of good x_1

Exercise 5 *Draw a graph and explain the intuition for the case where x_1 decreases but x_2 increases.*

Exercise 6 *Draw a graph and explain the intuition for the case where x_1 and x_2 decrease.*

Mathematically What happens as p_1 increases?

$$\frac{\partial x_1}{\partial p_1} = -\frac{\alpha m}{p_1^2} < 0$$

As price of good 1 increases, quantity demanded of good 2 decreases. Demand curve is negatively sloped. (law of demand)

1.3.2 Inferior good

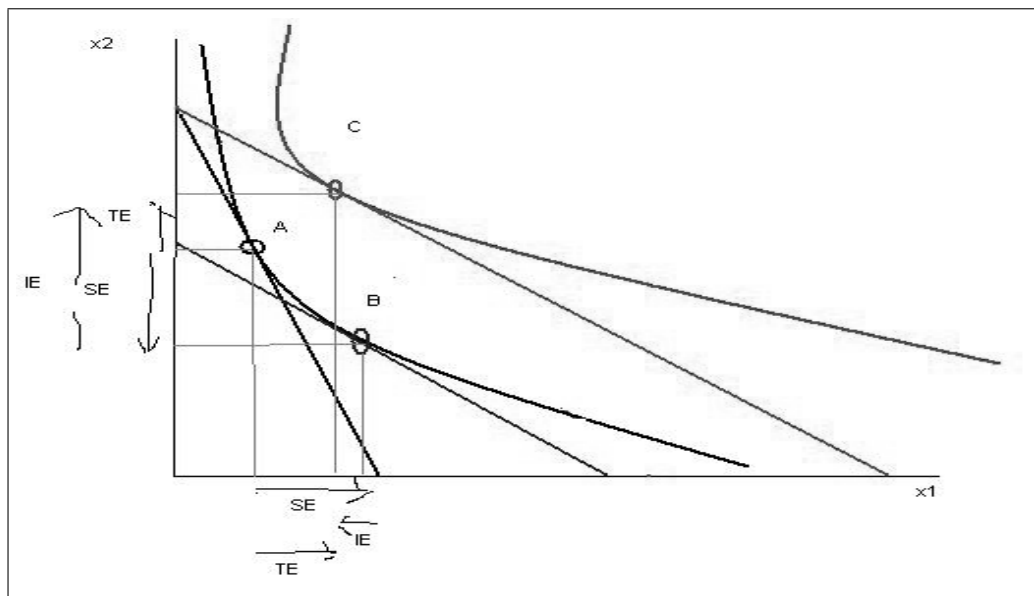
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1. Substitution effect: You substitute some from x_2 to x_1 : x_1 increases, x_2 decreases
2. Income effect: A decrease in p_1 is equivalent to an income increase. If x_1 is inferior and x_2 is normal then x_1 decreases and x_2 increases.

From the above we can see that with the information given above, the change in both x_1 and x_2 is ambiguous. Each depend on the relative magnitude of the substitution effect and the income effect.

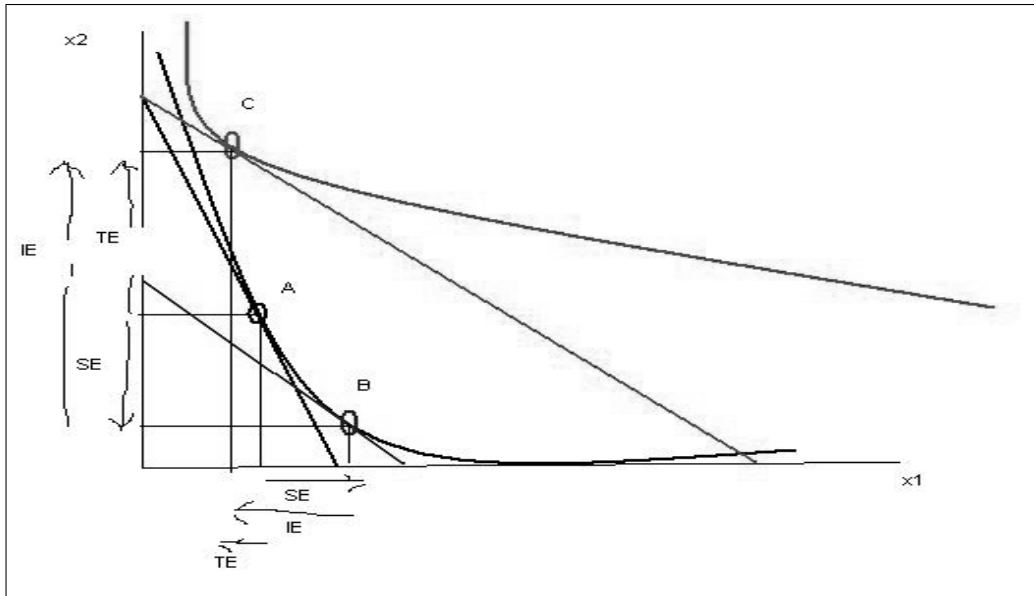
Lets consider the situation where for x_1 the substitution effect is larger than the income effect and for x_2 the income effect is larger than the substitution effect. In this case, we should see an increase in consumption of both x_1 and x_2 .

Figure 4: Decrease in p_1 when x_1 is inferior and $SE > IE$



When the $SE > IE$, we get the typical law of demand result: a decrease (increase) in price causes an increase (decrease) in the quantity demanded.

Figure 5: Decrease in p_1 when x_1 is inferior and $IE > SE$



Suppose that the $IE > SE$.

In this case, we see that we get the opposite result. A decrease in p_1 causes a decrease in x_1 ! The law of demand does not hold when $IE > SE$. It turns out that this peculiar situation is the case of a Giffen good.

Definition 7 *Giffen Good: as p_1 increases, x_1 increases*

The intuition behind a Giffen good is that as the price of some good increases which makes up a large part of your budget, it makes you so poor that you are forced to consume more of the good. The typical good discussed is the potato. The story goes that during the potato famine the price of potatoes increased dramatically. Potatoes were a staple of the Irish diet and essentially the cheapest good you could buy. When prices of potatoes went up, people were no longer able to afford their weekly meat portion (because more of their budget was being devoted to potatoes) so ended up consuming even more potatoes. In reality, evidence of a true Giffen good has not been found. For the moment, it remains a fascinating theoretical idea.

It turns out, for reasons we will see below, that in order for a good to be a Giffen good, it must be an inferior good. All Giffen goods are inferior goods but not all inferior goods are Giffen goods.

Exercise 8 *Draw and explain the case where p_1 increases and x_1 is an inferior but not Giffen good and x_2 increases.*

Exercise 9 Draw and explain the case where p_1 increases and x_1 is an inferior but not Giffen good and x_2 decreases

Exercise 10 Draw the case where p_1 increases and x_1 is a Giffen good and x_2 increases.

Exercise 11 Draw the case where p_1 increases and x_1 is a Giffen good and x_2 decreases

Here is a little summary chart of the above.

Type of good	Price Change	Income Effect	Substitution Effect	Total Change in Consumption
Normal	Price increases	x decreases	x decreases	x decreases
Normal	Price decreases	x increases	x increases	x increases
Inferior; Non-Giffen	Price increases	x increases	x decreases	x decreases
Inferior; Non-Giffen	Price decreases	x decreases	x increases	x increases
Inferior; Giffen	Price increases	x increases	x decreases	x increases
Inferior; Giffen	Price decreases	x decreases	x increases	x decreases

1.4 Applications

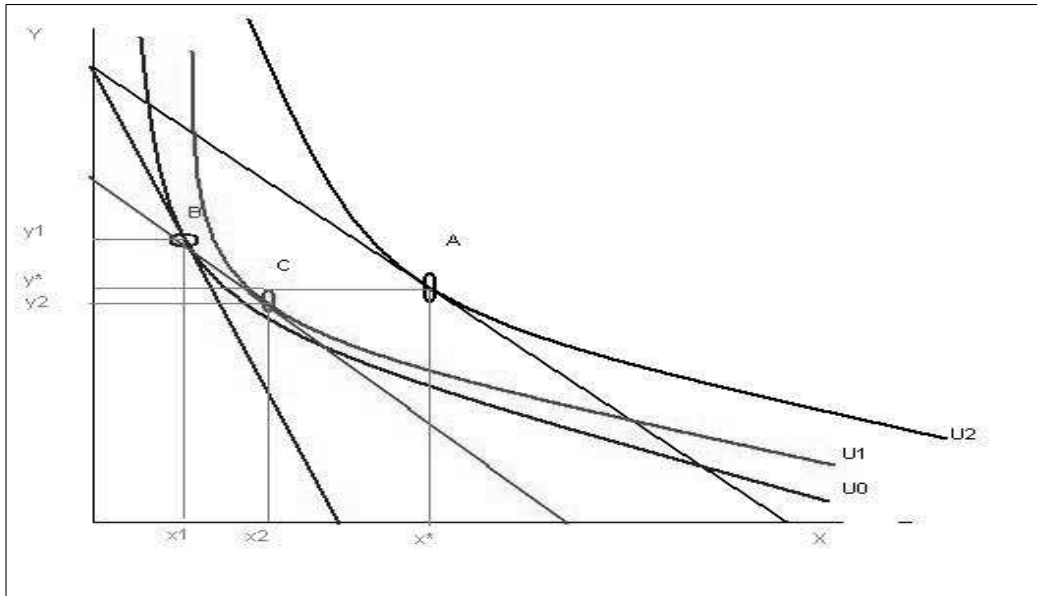
1.4.1 Lump-Sum Taxes

Definition 12 *The lump-sum principle says that taxes imposed on purchasing power will have smaller welfare costs than taxes imposed on a specific set of goods.*

Consider the graph below. Originally, if there is no government intervention, given the budget constraint,

$$m = p_x x + p_y y,$$

Figure 6: Lump Sum Taxes result in higher welfare relative to commodity tax



the individual would choose bundle A . Suppose the government imposes a per unit tax t on good x . This is equivalent to an increase in the price of x . The new budget constraint is

$$m = (p_x + t)x + p_y y.$$

At these prices the individual would choose bundle B . So the budget constraint holds for

$$m = (p_x + t)X_1 + p_y Y_1.$$

Note that the amount of tax revenue raised is tX_1 .

Suppose instead that a lump-sum tax was imposed. In other words, instead of taxing either good, the government will take away a certain amount of income such that the same amount of money is raised, tX_1 . In this case, there is no change in prices, just a change in income. So the new budget line must be parallel to the original budget line but go through bundle B . The new budget constraint is

$$m - tX_1 = p_x x + p_y Y$$

Note that these equations are the same only X_1, Y_1 . Therefore, they only intersect at bundle B . Note that with this new budget constraint, a higher utility level, U_1 , can be achieved at bundle C . This shows us that a higher utility level will be achieved through a lump sum tax than through a commodity tax when both taxes raise the same amount of revenue.