Game Theory

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Game theory is used to model economic situations in which agent (*player*) must make strategic choices and final outcomes depends on each players's decision. Can be used to model firm decisions in imperfect competition. Typically, we will just analyze the case of two players, player A and player B.

Each possible course of action is called a **strategy**. In a game, both player A and B must decide on a particular strategy. For example, in a game of firm competition, one strategy could be to spend a high amount on advertising; a second strategy could be to spend a low amount on advertising.

Returns at the end of a game are called the **payoffs**. For example, payoffs could be represented as the utility or profits that result from a certain outcome.

For the moment we assume full information; each player knows the other's pay

0.1 Nash equilibrium

A Nash equilibrium is a set of strategies (e.g. a^* =Firm A spends high on advertising, b^* =Firm B spends high on advertising) where a^* is player $A's$ best move when player B plays b^* and b^* is player B's best move when player A plays a^*. Each would stick with this strategy even if they knew with certainty what the other player would do. Not all games have a Nash equilibrium and some games have multiple equilibrium.

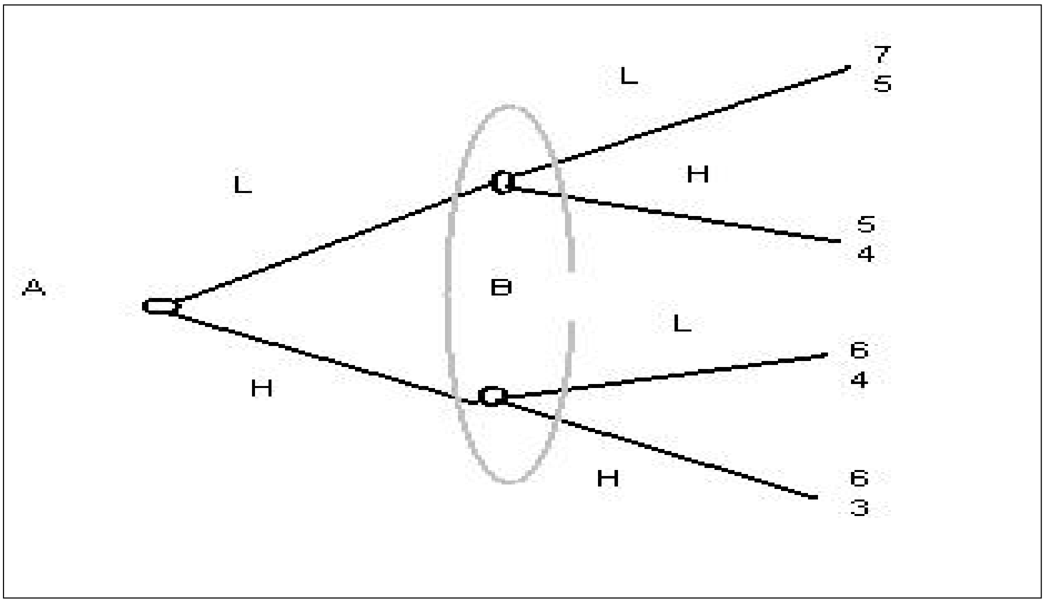
0.2 Dominant Strategy

A dominant strategy is one that you would choose regardless of which strategy the other player chooses.

1 Representation of a game

A game can be represented throught either a game tree (extensive form) or a game matrix (normal form). To understand these two forms, consider an example advertising game. Two firms, A and B, must decide how much to pay on advertising.

Figure 1: Stackleburg Model



1.1 Game Tree

In this game there are 2 players. This is denoted by A and B. The gray oval around B has two possible interpretations. The first is that A and B make their decisions at the same time. The second interpretation is that A goes first but that when B goes, it does not know A's decision. When A goes, it has two possible stratgies: adopt a high budget (H) or a low budget (L). If A chooses Low, firm B has the option to choose either Low or High. Similarly, if A chooses High, firm B has the option to choose either Low or High. Therefore, there are four possible outcomes:

- (Low, Low) in which case A gets a payoff of 7 and B gets a payoff of 5.
- (Low, High) in which case A gets a payoff of 5 and B gets a payoff of 4.
- (High, Low) in which case A gets a payoff of 6 and B gets a payoff of 4.
- (High, High) in which case A gets a payoff of 6 and B gets a payoff of 3.

A game tree is usually an easy way to conceptualize the problem.

1.2 Game Matrix

Just as with the game tree, one can see that each firm again has two possible strategies, H or L. There are 4 possible outcomes. To read this matrix, consider the intersection of two strategies. The matrix tells us that if both A and B play *Low*, A will have a payoff of 7 and B will have a payoff of 5.

Table 1: Game in Matrix form

		B's Strategies	
		L	**H**
A's Strategies	L	$7, 5$	$5, 4$
	H	$6, 4$	$6, 3$

A game matrix is typically an easy way to solve the game.

Lets first determine if either player has a dominant strategy. We can see that regardless of whether A plays L or H, B will always do better by playing L. A does not have a dominant strategy. However, since A knows that B will play L, A will also play L. Therefore, the equilibrium for this game is L, L. Note that this is a Nash Equilibrium. L is $A's$ best strategy if B plays L and L is $B's$ best strategy if A plays L.

Example 1

Table 2: Rock, Scissors, Paper

		B's Strategies		
		Rock	**Scissors**	**Paper**
	Rock	$0, 0$	$1, -1$	$-1, 1$
A's Strategies	Scissors	$-1, 1$	$0, 0$	$1, -1$
	Paper	$1, -1$	$-1, 1$	$0, 0$

Example 2

Table 3: Battle of the Sexes

		B's Strategies	
		Mountain	**Seaside**
A's Strategies	Mountain	$2, 1$	$0, 0$
	Seaside	$0, 0$	$1, 2$

2 The Prisoner's Dilemma

The Prisoner's Dilemma is a famous game with a number of different applications. The basic scenario is as follows. Two people are arrested for a crime. The DA doesn't have any real evidence but is certain that they committed the crime. Thus, she tries to create situation where it will be optimal for the two to confess. She separates the two and gives each of them the following proposal. If you confess and your partner doesn't, you will get a reduced sentence of 6 months in jail for cooperating. Your partner will get 10 years. If you both econfess, you will each get 3 years in jail. If nether confesses, the DA has enough evidence to get them convicted of a lesser crime, which will result in a 2 year sentence for each. We can express this in a game matrix.

Table 4: Prisoner's Dilemma

		B's Strategies	
	Confess	**Not Confess**	
A's Strategies	Confess	3 years,3 years	6 months,10 years
	Not Confess	10 years,6 months	2 years,2 years

The way the DA has structured the game, each player has a dominant strategy of confessing, since regardless of what the other does, each will receive a lower jail time. Thus, each will confess and each will end up with a 2 year prison term. In reality, it would be better for neither to confess since they would each serve a lower prison time. But, this outcome isn't stable; each has an incentive to confess. Thus, the prisoner's dilemma.

This result is only for the case of playing the game once. If the game was played multiple times the outcome could be different.

2.1 Applications

Example 3 *Advertising game: Should each firm spend a lot or a little on advertising? The outcome*

Table 5: Advertising Prisoner's Dilemma

		B's Strategies	
	L	**H**	
A's Strategies	L	7,7	3,10
	H	10 ,3	5 ,5

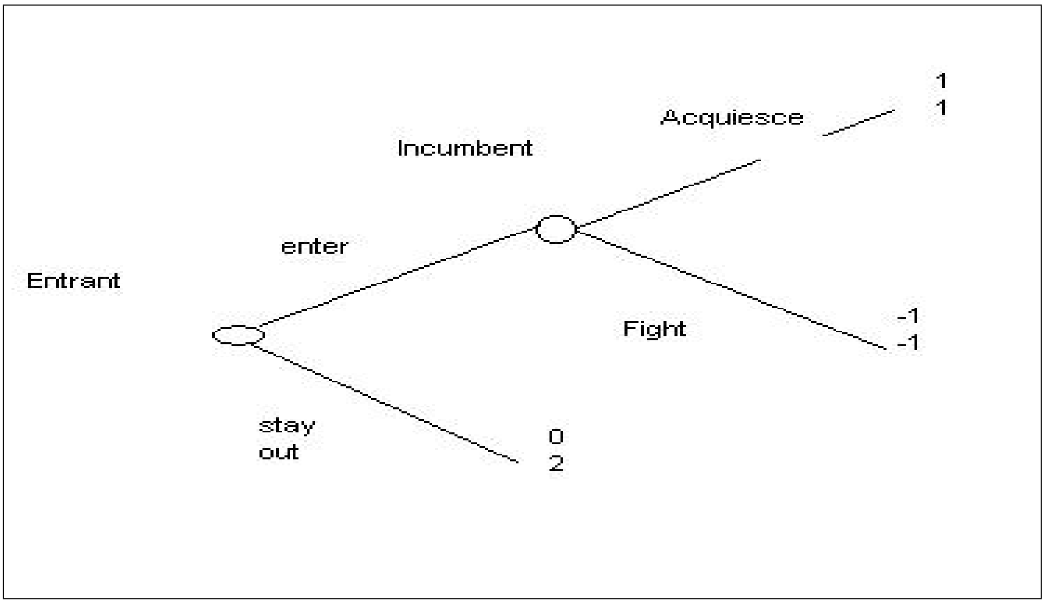
is H,H but the firms would prefer the outcome L,L. But, this is unstable - if each firm knew that the other would do low spending on advertising, each would have an incentive to cheat and do high spending.

Similar examples include frequent flyer miles and OPEC.

3 Sequential Games

So far, we have assumed that two players make their moves simultaneously or that play sequentially without knowing what the other player has done. In fact, there are many cases where action is sequential and the second player knows what the first player has done. Chess or checkers would be examples of this. In economics, consider the following situation. One firm is currently producing in a certain market (the incumbent). A second firm is considering whether or not to enter this market

Figure 2: Entry Game



(the entrant). If the entrant stays out, the status quo prevails and the game is over. If the entrant enters, the incumbent has two possibilities: fight by lowering the price or acquiesce and do nothing.

In the case of a sequential game, we can solve it by backwards induction. The basic idea here is that we can figure out what the second player's best moves will be for either of the first player's moves and from this figure out what the best move for the first player would be.

We can solve the above game. The Entrant can determine that the Incumbent acquiese if she enters (because $1 > -1$). It is not a credible threat for the Incumbent to say that he will fight. Therefore, the Entrant will enter (because $1 > 0$).

Suppose A played L. Then B would play L as it gives them a better outcome. Suppose A played H. Then B would play L. Of the two, A prefers the outcome of 7 so A would choose to play L. Therefore, the strategy profile will be

$$
\begin{aligned}
A &: \textit{Low} \\
B &: \textit{Low if low} \\
&: \textit{Low if high}
\end{aligned}
$$

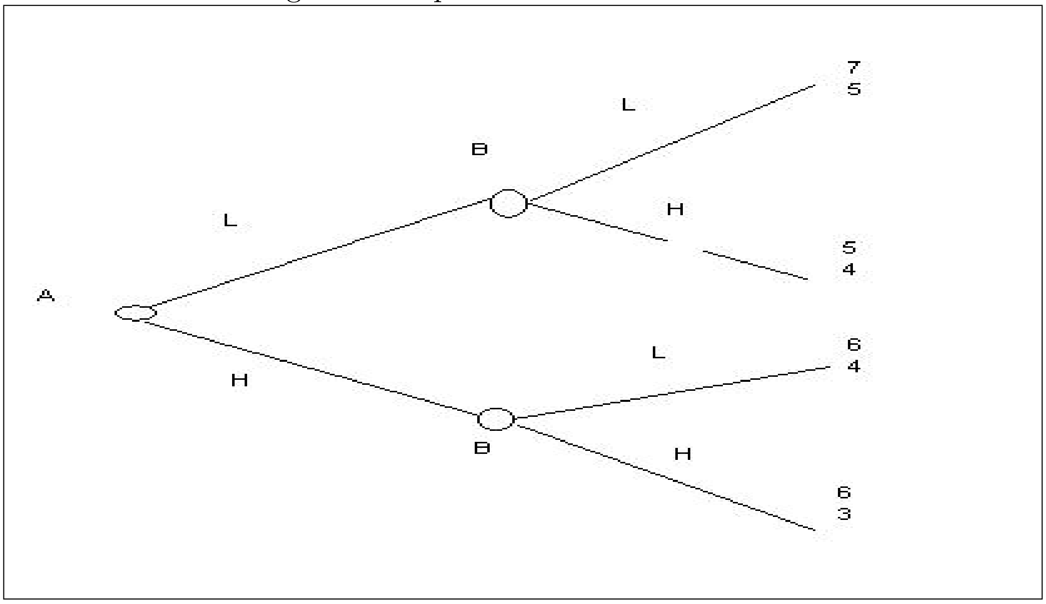
4 Bertrand Model

4.1 One-Time play

Consider the case where you have two firms producing goods that are perfect substitutes for each other and where each firm has the same marginal costs, c. For simplicity, assume $ATC = MC$ (ie,

Figure 3: Seqential Game

Example 4



no fixed costs). Because the goods are perfect substitutes, customers will buy from the firm with the cheapest price. If both firms have the same price, then demand is split evenly between the two firms.

Note that in this kind of model, the dominant strategy is to lower your price. For example, if you knew for certain that the other firm set $P > c$, you have an incentive to lower your price because you could capture the entire market and thus increase your profit. On the other hand, if you knew that the other firm was going to lower its price, you would also have an incentive to lower your price; otherwise you would sell nothing. Therefore, each firm will choose to lower its price. If we take this thinking to its logical conclusion, each firm would lower its price until $P_A = P_B = c$. No firm would lower its price below this because then it would earn negative profits. So $P_A = P_B = c$ is a Nash Equilibrium because firms have no incentive to change the price. Thus, in the Bertrand equilibrium, we get the perfectly competitive result that $P = ATC$. Collusion is not possible in this model.

4.2 Finite Time Horizon

What if the Bertrand game was repeated several times and each firm knew that time period in which the game would end. Is collusion possible then? For explanatory purposes, suppose the game was to be played twice. In period 2, each firm would have an incentive to cheat and lower its price; there is no incentive to collude in the last period because you know you can't be punished for cheating. From our logic above, we know that each firm would lower its price until the point $P_A = P_B = c$. What should a firm do in period 1, knowing that in period 2 collusion in not possible and $P_A = P_B = c$? Again, there is an incentive to cheat; why not when there can be no punishment in period 2? So, we get the same result, $P_A = P_B = c$. The result is the same, regardless of the number of time periods, if you know in which period of time it will end. Note that we are solving this game using backwards induction.

In the case of an infinite time horizon, collusion is possible. It turns out to depend on the interest rate.