Costs

March 23, 2004

1 Economic profits and cost minimization

1.1 Costs

Assume that inputs are hired in perfectly competitive markets. Firms can hire all the capital or labor they want at the current rates; how much they buy does not affect the rental prices.

$$\begin{aligned} \text{Total Costs} &= TC = wL + rK \\ w &= \text{hourly wage} \\ r &= \text{hourly rental rate for capital} \end{aligned}$$

1.1.1 Isocost Curves

We want to graph costs. An isocost curve shows all the combinations of L and K that give a certain cost. Graphically, an isocost curve looks like a budget constraint, as it will be a straight line. Solving for K,

$$K = \frac{TC - wL}{r}$$

we see that the slope is $-\frac{w}{r}$ (the ratio of the input prices.) Changes in the total cost will shift the isocost curve out. Higher costs are associated with isocost curves that are further out from the origin.

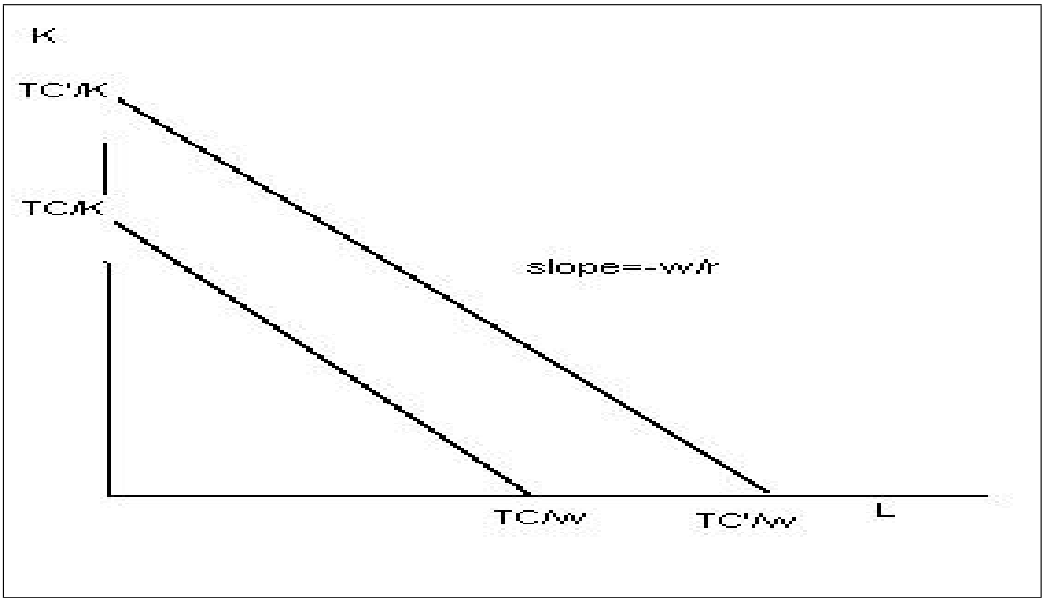
1.2 Economic Profits

Define economic profits (π) as:

$$\begin{aligned} \pi &= \text{Total revenue } (TR)\text{-Total costs} \\ &= pq - wL - rK \\ &= pf(L, K) - wL - rK \end{aligned}$$

Recall that TC includes both implicit and explicit costs.

Figure 1: Iscosost Functions



2 Cost-Minimizing Input Choice

Suppose for some reason that a firm had decided to produce output level q_1. Once this decision has been made, TR is fixed. Profit will be maximized when TC are minimized. Given this production level, the firm needs to choose the input combinations L and K that will result in the lowest cost.

Lets first look at the cost-minimizing choice graphically and see what intutition it gives us. We know that the firm had decided to produce q_0. It wants to do so at the lowest possible cost. The lowest isocost curve (TC_0) shown on the graph will not allow the firm to produce at q_0 so it isn't an option for the firm. Either TC_1 or TC_2 would allow the firm to produce q_0 (i.e., points A and B). Given that, the firm would obviously decide to produce at B rather than A because it is a lower cost. We see that at B or L^*, K^*, the slopes of the isoquant and the isocost are equal:

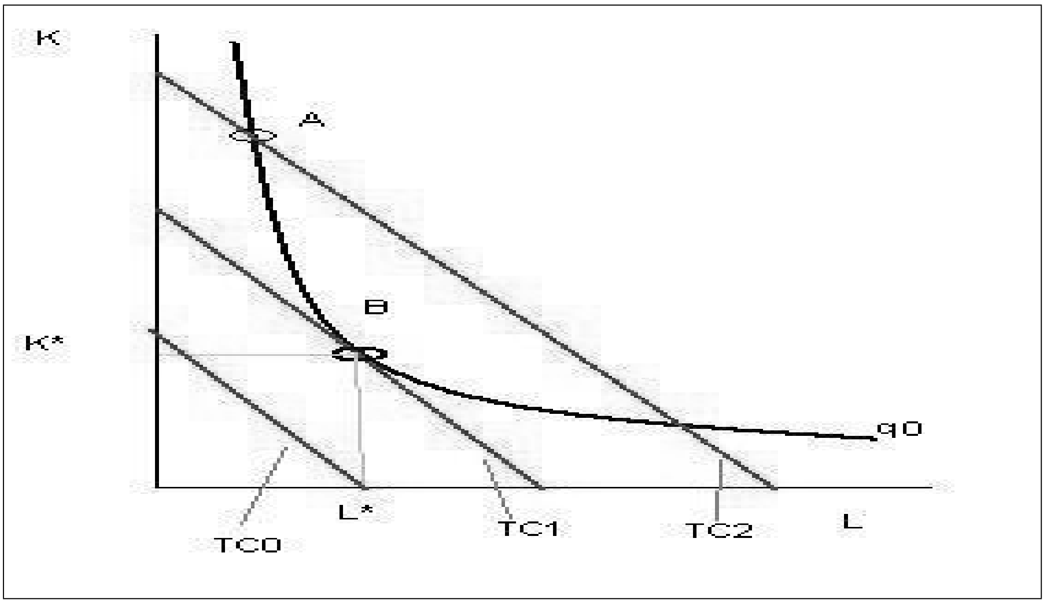
$$-\frac{w}{r} = MRTS.$$

Rewriting this slightly,

$$\begin{aligned} -\frac{w}{r} &= -\frac{MP_L}{MP_K} \\ \frac{w}{r} &= \frac{MP_L}{MP_K} \\ \frac{MP_L}{w} &= \frac{MP_K}{r}. \end{aligned}$$

Note that $\frac{MP_L}{w}$ represents the marginal increase in output from an additional worker divided by the price of the worker. In other words, $\frac{MP_L}{w}$ is the marginal output per dollar spent on labor. $\frac{MP_K}{r}$ is

Figure 2: Cost Minimization



the marginal output per dollar spent on capital. At the cost-minimizing input levels, the marginal output per dollar spent is the same, regardless of whether one uses labor or capital to produce this additional outuput. Contrast this with point A. Here, the slope of the isoquant is greater than the slope of the indifference curve.

$$\begin{aligned} MRTS &> -\frac{w}{r} \\ -\frac{MP_L}{MP_K} &> -\frac{w}{r} \\ \frac{MP_L}{MP_K} &> \frac{w}{r} \\ \frac{MP_L}{w} &> \frac{MP_K}{r}. \end{aligned}$$

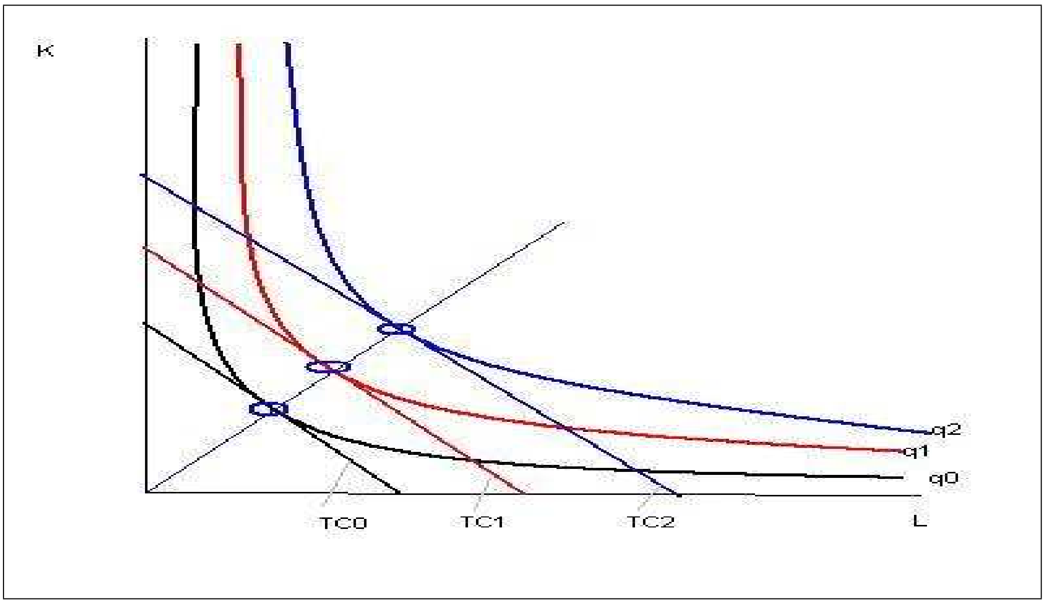
So at A, the marginal output per dollar spent on labor is greater than the marginal output per dollar spent on capital. Any smart firm would realize that one could have lower costs in this case by using more labor and less capital. That is exactly what we see when we consider the move from A to B.

3 Cost Curves

3.1 Firm's Expansion Path

We just figured out for a given output level what the optimal level of inputs would be. Imagine repeating that process several times, for several different output levels. By repeating this process, we can figure out the cost minimizing input levels for different levels of output.

Figure 3: Expansion Path



3.2 Total Cost Curve

Now imagine plotting the quantity produced versus different costs. This graph would show the cheapest cost for all possible output levels.

Example 1 *Suppose in our graph above,* $q_0 = 10, q_1 = 20, q_2 = 30$ *and that* $TC_0 = 10, TC_1 = 20, TC_2 = 30$. *Graph the cost minimizing Total Cost function. Obviously, it is a straight line with slope of 1.*

In fact, how these total cost curves look depends on the returns to scale. The graph below shows four possible ways of drawing the TC curve. Let's think about why each makes sense, given the different definitions of RTS. The first graph shows CRTS. Recall that CRTS says that when we increase the inputs by a factor t, output increases by a factor t. For costs, if we increase inputs by a factor t,

$$\begin{aligned} w(tL) + r(tK) &= t(wL + rK) \\ &= t * TC, \end{aligned}$$

TC increase by a factor t. So with CRTS, if an increase in inputs causes output to increase by a factor t, it will also cause costs to increase by a factor t.Therefore, we see that when a production function exhibits CRTS, whatever proportion you increase inputs by, output and costs will increase

Figure 4: Cost Minimizing Total Cost Function

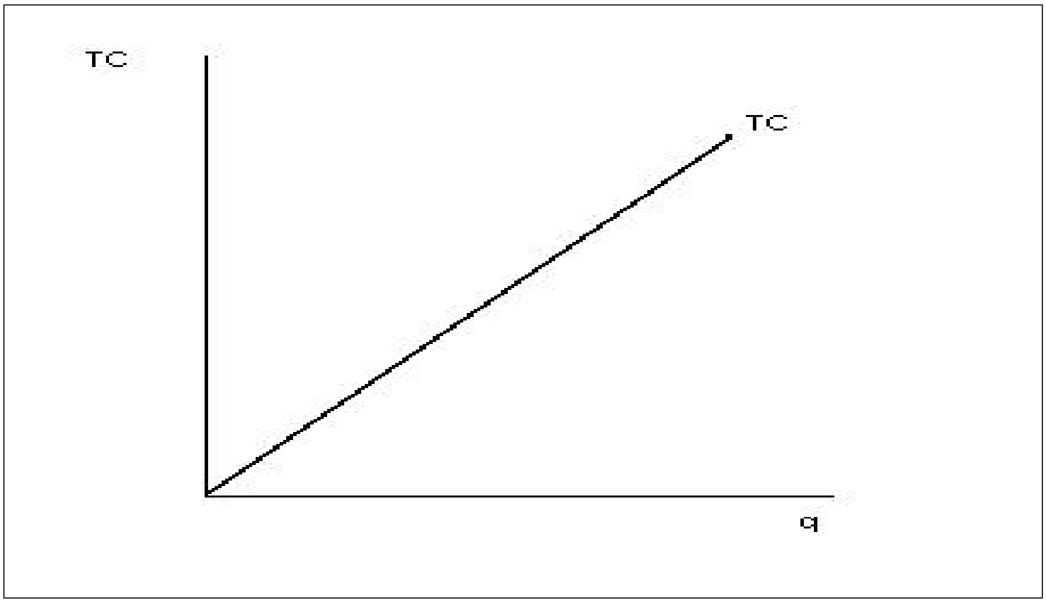
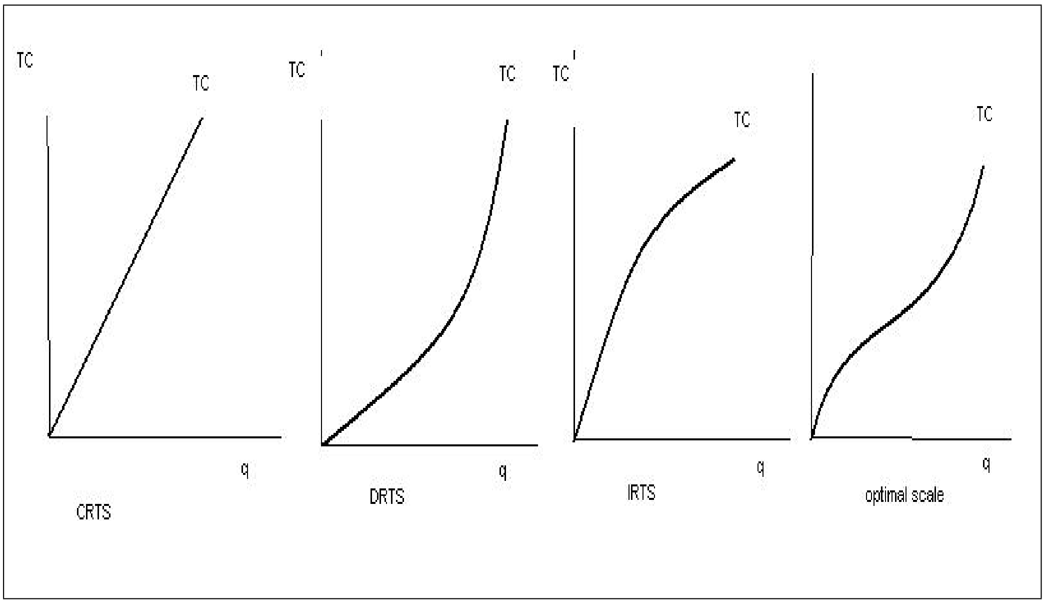


Figure 5: Shapes of Cost Functions



by the same proportion. So each increase in output has a proportional increase in cost; therefore, the TC line must have a constant slope.

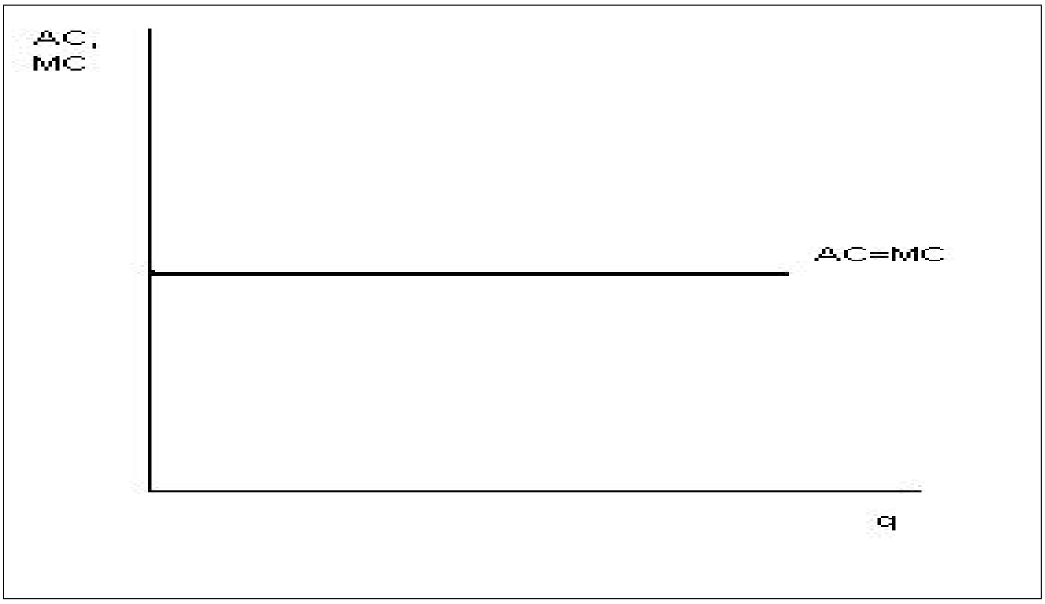
Similarly, with IRTS, doubling inputs causes output to more than double and costs to double. So the TC line must be concave. With DRTS, doubling inputs causes output to less than double but costs to double. So the TC from a DRTS production function will be convex. Finally, it is possible that the production function exhibits different RTS over differnet levels of output. It might have IRTS at the beginning and then DRTS. Our TC function will look like a combination of the IRTS and DRTS TC functions.

3.3 Marginal Cost and Average Cost

We're often interested in the marginal and average costs. The MC function tells us how costs change as output changes. The AC function tells us the per unit cost for each unit.

$$AC = \frac{TC}{q}$$
$$MC = \frac{dTC}{dq}$$

Figure 6: AC and MC for CRTS



AC and MC are both functions of q. To understand better the relationship between the two, we can examine how the two functions differ with q. Lets take the derivative of the AC function with respect to q. Note that

$$AC(q) = \frac{TC(q)}{q}.$$

We will need to apply the quotient rule (derivative of the top times the bottom minus the derivative of the bottom times the top all over the bottom squared).

$$\begin{aligned}
\frac{dAC}{dq} &= \frac{\frac{dTC}{dq} * q - TC}{q^2} \\
&= \frac{MCq - TC}{q^2} \\
&= \frac{MC}{q} - \frac{AC}{q} \\
&= \frac{MC - AC}{q}.
\end{aligned}$$

So, we see that in general, the slope of the average cost function will be determined by the relationship between AC and MC.

Note that for the first unit, the AC and MC must be the same.

Lets consider the possible cases. First, suppose that over all q, the AC has a positive slope. This implies that $MC > AC$. Therefore, MC must also be positively sloped. If MC is positively sloped it means that each unit you produce is more expensive then the last. This occurs when you have $DRTS$.

Now suppose that over all q, the AC has a negative slope. This implies that $MC < AC$. Therefore, MC must also be negatively sloped. If MC is negatively sloped it means that each unit you produce is cheaper then the last. This occurs when you have $IRTS$.

Now suppose that over all q, the AC has 0 slope. This implies that $MC = AC$. Therefore, MC must also have 0 slope. If MC has 0 slope it means that each unit you produce costs exactly the same as the previous unit. This occurs when you have $CRTS$.

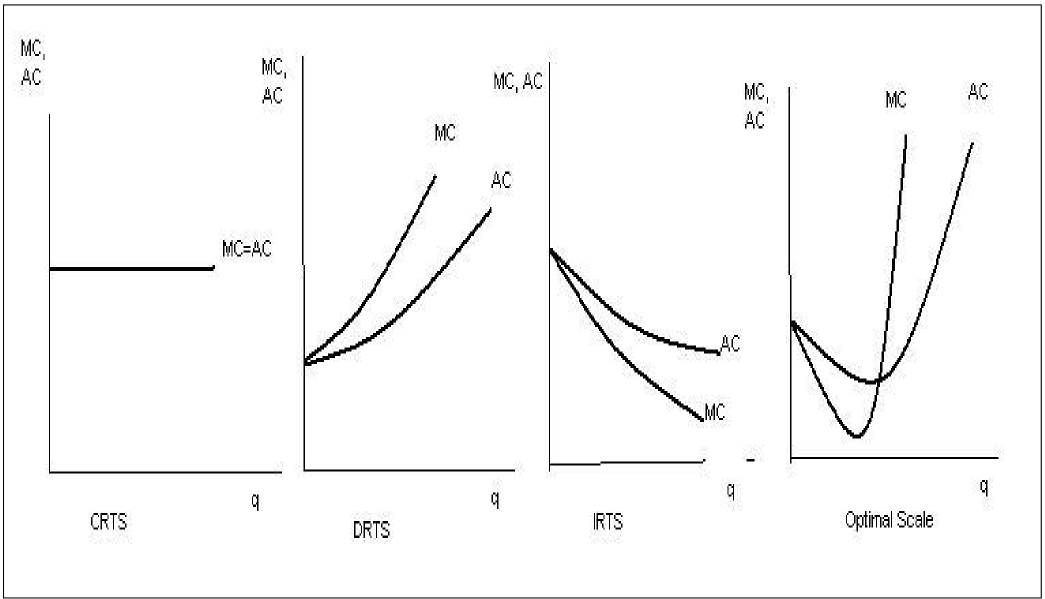
The last possible case is that the slope of AC has different slopes. This occurs with optimal scale when the production function has different returns to scale. A firm would start out with $IRTS$, then $CRTS$, and finally $DRTS$. Note that MC must intersect AC at its lowest point. We know this because when $MC = AC$, then $\frac{dAC}{dq} = 0$. This says that at this point, the slope of the AC function is 0; this only occurs at a minimum or maximum.

3.4 Example

Suppose $TC = q^3 - 40q^2 + 430q$

Sketch the general shape of the TC function. (Hint: a function is concave if its second derivative is negative and convex if its second derivative is positive.) Calculate and sketch AC. At what point

Figure 7: MC and AC



does it reach a minimum?

Calculate and sketch MC.

3.5 Empirical Average Costs

The table below shows average costs for a variety of industries. The numbers represent the costs for different sized firms asa percentage of the minimal average cost.

Industry	**Small**	**Medium**	**Large**
Aluminum	166.6	131.3	100.0
Automobiles	144.5	122.7	100.0
Electric Power	113.2	100.0	101.5
HMOs	118.0	106.3	100.0
Hospitals	129.6	111.1	100.0
Life Insurance	113.6	104.5	100.0
Lotteries (State)	175.0	125.0	100.0
Sewage Treatment	104.0	101.0	100.0
Trucking	100.0	102.1	105.6

Table 1: Long Run Average Cost Estimates Based on Firm Size

4 Long Run versus Short Run

In the long run all inputs are variable. This is what we have examined so far. In the short run, at least one input is fixed. This implies that the costs for this fixed input are also fixed. Thus, in the short run we have fixed costs and variables costs.

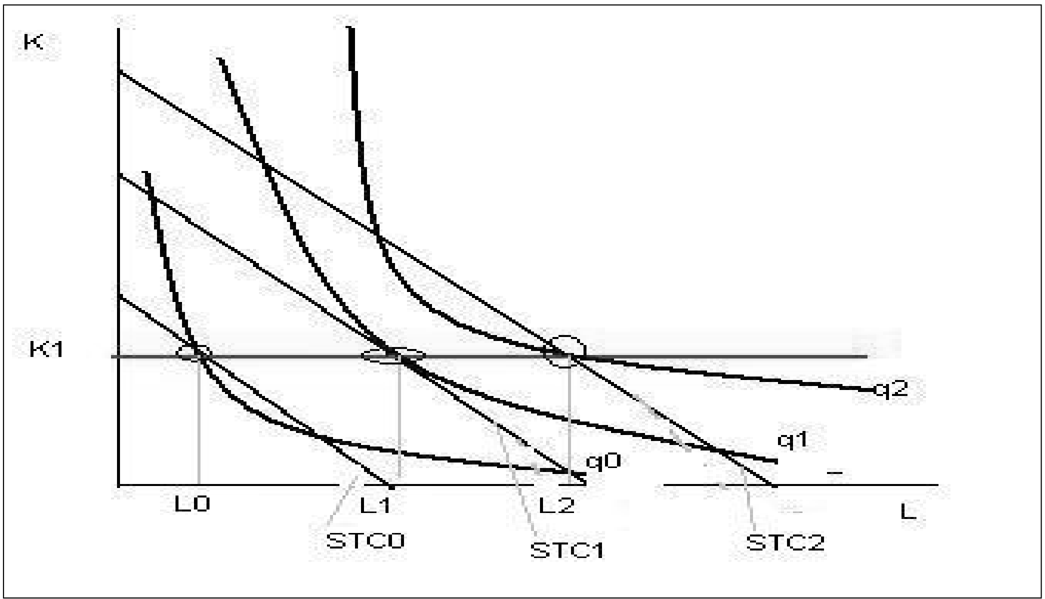
4.1 Cost Minimization in the Short Run

Suppose that capital is fixed in the short run. Given this, the firm only has the flexibility to choose the amount of labor to use. In fact, if capital is fixed and the firm wants to produce a certain level of output, the amount of labor to use is already predetermined and it may not be 'optimal'.

Consider the graph below. Capital is fixed at K_1. If the firm wants to produce q_0, it must use L_0. Graphically, we can see that this is not optimal. At this point, the slope of the isoquant is greater than the slope of the isocost curve. This means that $\frac{MP_L}{w} > \frac{MP_K}{r}$ or that per dollar, labor will contribute a larger increase in output than capital. Therefore, it would be more cost-effective to use more labor and less capital to produce output level q_0. But, we can't because the level of capital is fixed.

Looking at the graph, we see that there is only one point where the input combination is optimal. At all other points, non-optimal levels are being used. Lets translate this into cost terms. We know that in the LR optimal (cost-minimizing) input levels are used for every output level. In the SR, there is only one output level at which the optimal inputs will be used. At all other output levels, non-optimal inputs will be used. Therefore, there is one output level in the SR when the costs will be truly minimized and therefore the same as the long-run costs. At all other output levels, the costs

Figure 8: Inputs in the Short Run



in the SR will be higher than the costs in the LR. Graphically, this means that the short-run average cost curve (SAC) will be higher than the long-run average cost curve (LAC) at all output levels but one. From another perspective, we know that the costs are different in the long run and short run.

$$\begin{aligned} TC &= wL^* + rK^* \\ STC &= wL + r\overline{K} \end{aligned}$$

These will only be equal when $\overline{K} = K^*$ and $L = L^*$. Given this, the average cost and marginal cost functions for the short run and long run must be different. Using the same logic as before, we know that the SMC must intersect the SAC at its lowest point.

4.2 SR versus LR example

Suppose $q = KL$. Suppose $w = 5$ and $K = 5$.

5 Shifts in Cost Curves

Cost curves shift when factors in our underlying model affect the optimal inputs. Two things that cause shifts are:

- Changes in input prices: When input prices change, this causes a pivot in the isocost curve and will result in new optimal inputs being chosen. If you repeat this for several different production levels at the new prices, you will generate a new expansion path and thus a new TC curve.

Figure 9: Short Run and Long Run Costs

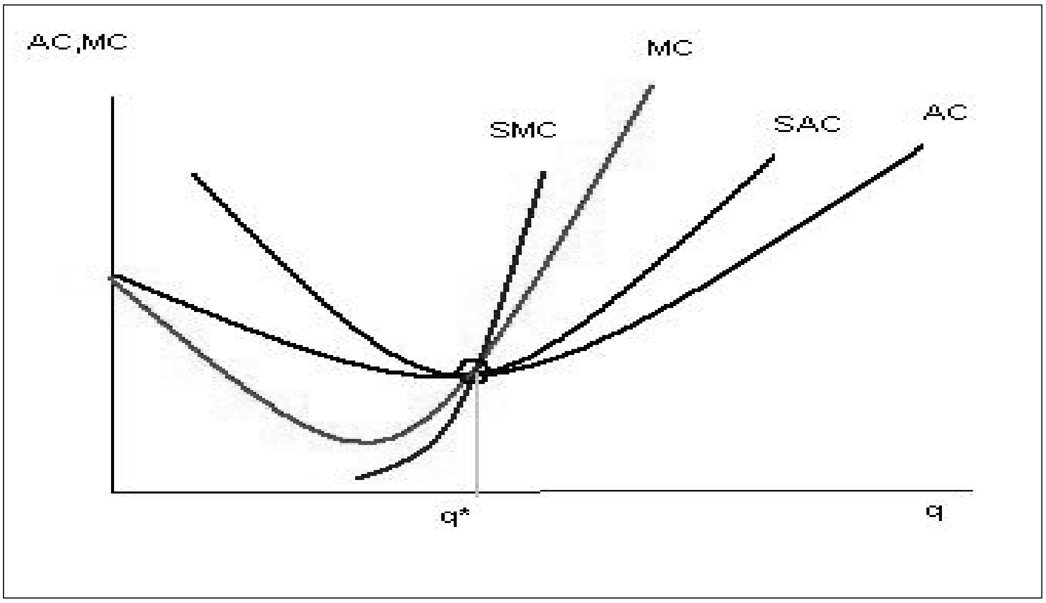
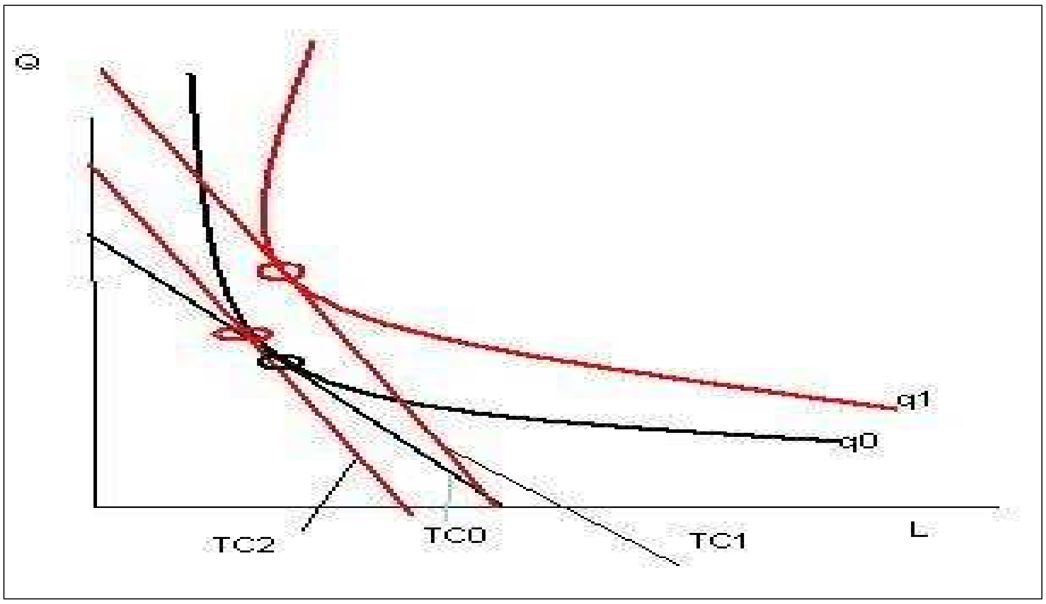


Figure 10: Change in input prices affect isocost curves



- Technological changes: If the technology that is used to produce the good changes, this will cause the production function to change. This in turn will result in new isoquants. From the same logic as above, new isoquants result in new expansion path and a new TC curve.