

# 1 Choice: The Graphical Approach

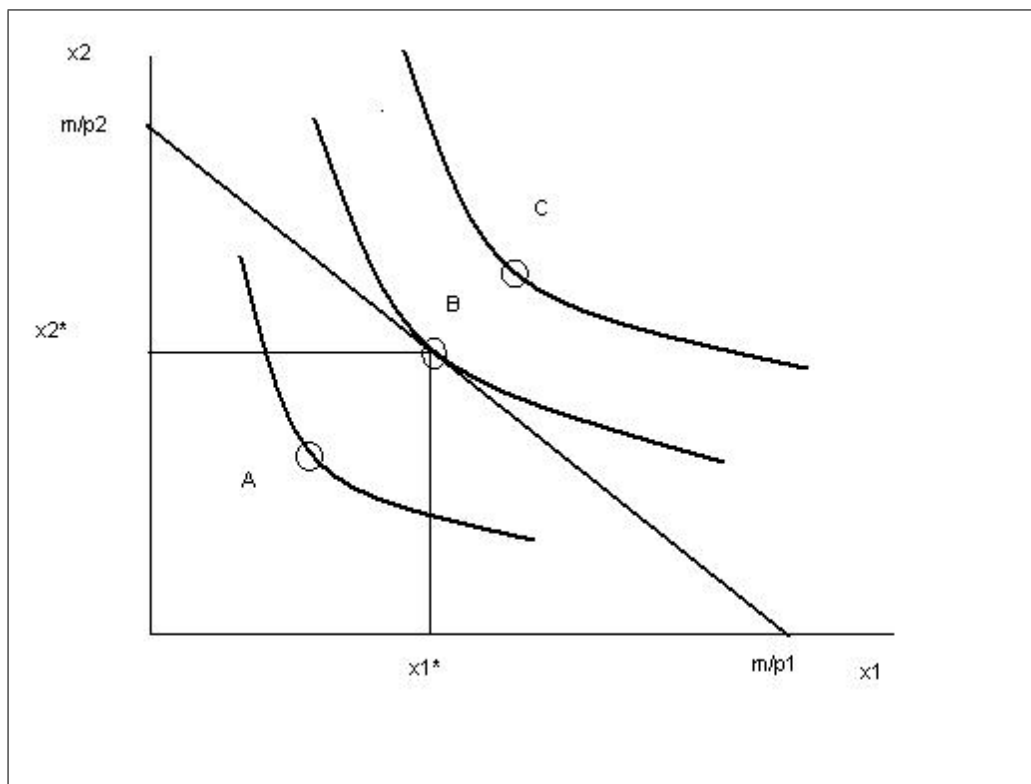
At last! Having studied consumer preferences and constraints, we're ready to apply our model.

Recall model: Individuals choose best bundle they can afford.

Later we'll come back to this mathematically.

Recall: budget line shows what you can afford; ic shows your preferences.

Our model says that we want to find the highest ic within reach of our budget.



Would choose bundle B. Why?

- Can afford bundle better than A.
- Would most like to be on the indifference curve with  $U = 3$  - consuming any bundle on this indifference curve would give you a higher utility level than on  $U = 2$  or  $U = 1$ .
- But, what is affordable? By looking at the budget constraint, we see that we can afford bundle B and bundle A. We can't afford bundle C.
- Of the two bundles, we can afford, we prefer bundle B to A because B is associated with a higher utility level (indifference curve is further out)
- We say that bundle B,  $(x_1^*, x_2^*)$ , is optimal bundle

Note: Graphically, we can see that at the optimal bundle, budget line is tangent to indifference curve

- slope of ic=budget line

$$MRS = -\frac{p_1}{p_2}$$

## 1.1 Interpretation 1

MRS is the rate at which you are willing to trade off between the two goods.

$\frac{p_1}{p_2}$  is the rate at which you can trade off between the two goods in the marketplace. The slope of this line is  $-\frac{p_1}{p_2}$ . If give up one unit of  $x_1$  get  $\frac{p_1}{p_2}$  additional units of  $x_2$ .

So,

$$MRS = -\frac{p_1}{p_2}$$

says that when you are maximizing, the rate at which you are willing to trade off between the two goods equals the rate at which the market is willing to trade-off between the two goods.

Suppose this wasn't the case. Suppose

$$|MRS| > \left| -\frac{p_1}{p_2} \right|$$

at your consumption level. Graphically, that looks like the following:

We can see that if  $MRS > \frac{p_1}{p_2}$  in absolute value then you wouldn't choose to consume bundle A. At A, the slope of the indifference curve (MRS) is greater than the slope of the budget constraint ( $-\frac{p_1}{p_2}$ ). We can see that there is another bundle, B, that you can afford and which is better than A because it is on a higher indifference curve. So, graphically we see that  $MRS > -\frac{p_1}{p_2}$  is not possible.

Why isn't it possible intuitively? If  $MRS > -\frac{p_1}{p_2}$ , it says that I'm willing to trade more of  $x_1$  for  $x_2$  than it would cost me to do that. For example, suppose  $MRS = -4$  and  $\frac{p_1}{p_2} = -2$ . One interpretation of this is that I'm willing to trade 4 units of  $x_2$  in order to get 1 unit of  $x_1$  but that in the market anyone will give me 1 unit of  $x_1$  in exchange for just 2 units of  $x_2$ . Obviously I would take the trade from the market. Therefore, consuming the point where

$$|MRS| > \left| -\frac{p_1}{p_2} \right|$$

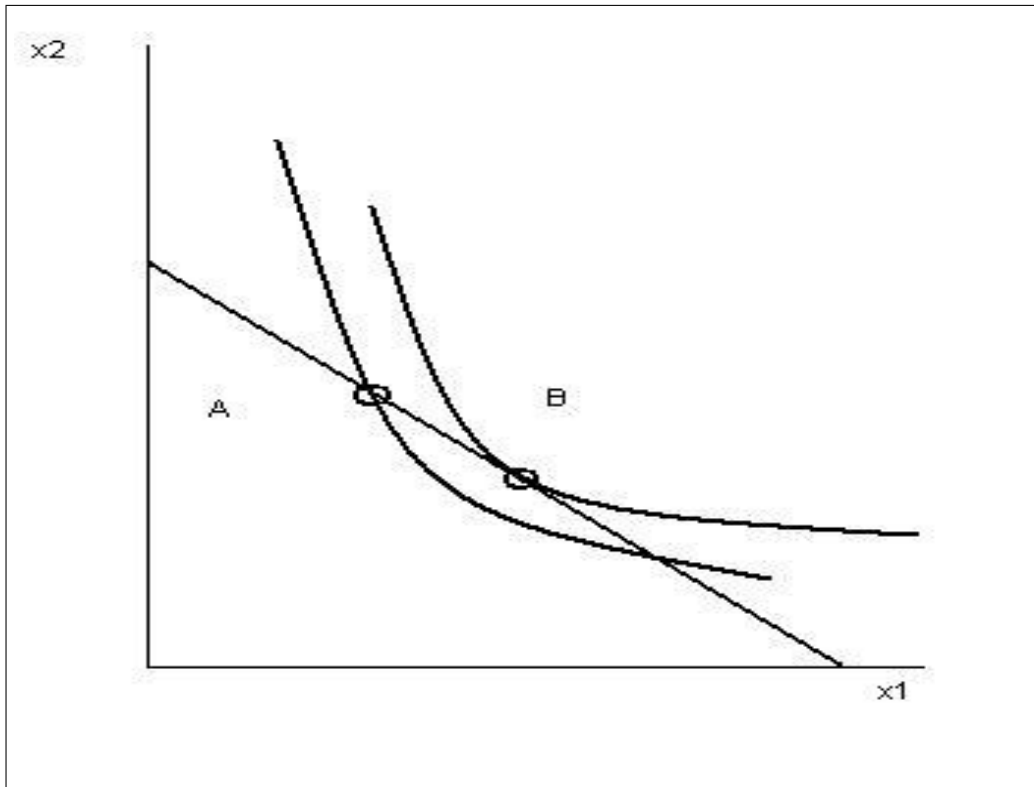
cannot be an equilibrium. Similarly, suppose

$$|MRS| < \left| -\frac{p_1}{p_2} \right|$$

and that the same numbers apply. Then, I'm willing to trade 2 units of  $x_2$  for 1 unit of  $x_1$ ; in the market 1 unit of  $x_1$  trades for 4 units of  $x_2$ . I would sell in the market so therefore,

$$|MRS| < \left| -\frac{p_1}{p_2} \right|$$

cannot be an equilibrium.



## 1.2 Interpretation 2

We can get a second interpretation of the utility maximizing condition by reinterpreting MRS. To do that, we first need to introduce a new concept.

### 1.2.1 Marginal Utility

Recall: marginal=small change

**Definition 1** *MU: change in utility when change consumption of one good a little bit*

$$MU_1 = \frac{\Delta U(x_1, x_2)}{\Delta x_1}$$

$$MU_2 = \frac{\Delta U(x_1, x_2)}{\Delta x_2}$$

Later, we'll define this concept more precisely using calculus.

The earlier assumptions we made about preferences leads to the fact that we will have diminishing MU - the more you consume of something, the smaller the increase in your utility. Think back to the ginger cookie example.

ex) Drink of water.

### 1.2.2 Marginal Rate of Substitution (again)

Recall, that this represents the change in the consumption of  $x_1$  and  $x_2$  that keeps utility constant. Can interpret using  $MU$ .

$$\begin{aligned}\Delta U &= \Delta x_1 MU_1 + \Delta x_2 MU_2 = 0 \\ \frac{\Delta x_2}{\Delta x_1} &= -\frac{MU_1}{MU_2} \\ MRS &= -\frac{MU_1}{MU_2}\end{aligned}$$

Note that by the assumption of more is better,  $MU_1 > 0$  and  $MU_2 > 0$ ; therefore, MRS, the slope of the indifference curve, must be negatively sloped.

### 1.2.3 Utility maximization condition

Recall that the graphical condition for maximizing utility subject to a budget constraint is:

$$MRS = -\frac{p_1}{p_2}$$

Rewriting this, we get:

$$\begin{aligned}-\frac{MU_1}{MU_2} &= -\frac{p_1}{p_2} \\ \frac{MU_1}{MU_2} &= \frac{p_1}{p_2} \\ \frac{MU_1}{p_1} &= \frac{MU_2}{p_2}\end{aligned}$$

This rewritten form the utility maximization condition has a very nice interpretation. Consider the first term,  $\frac{MU_1}{p_1}$ . Literally translated, this is "the small increase in utility from consuming a little more  $x_1$ , divided by the price of good 1". Or, "how much my utility would increase if I spent an additional \$1 on good 1". So, the utility maximization condition,

$$\frac{MU_1}{p_1} = \frac{MU_2}{p_2},$$

says that utility is maximized when the increase in utility per dollar is the same for both goods. To see why this makes sense, consider what would happen if they were not equal. (This is the form of a 'proof by contradiction'.) Suppose

$$\frac{MU_1}{p_1} > \frac{MU_2}{p_2}.$$

This says that my utility would increase by more if I spent an additional dollar on  $x_1$  than if I spent it on  $x_2$ . So being rational, what would you do? You'd spend your extra dollar on  $x_1$ . What happens

as you buy more  $x_1$ ? By diminishing MU,  $MU_1$  decreases. Since you have a limited budget, if you are spending more on  $x_1$  you must be spending less on  $x_2$ . Therefore, by diminishing MU,  $MU_2$  increases. This process will continue until the two ratios are equal.

On your own, you should go through the opposite logic (when  $\frac{MU_1}{p_1} < \frac{MU_2}{p_2}$ ).

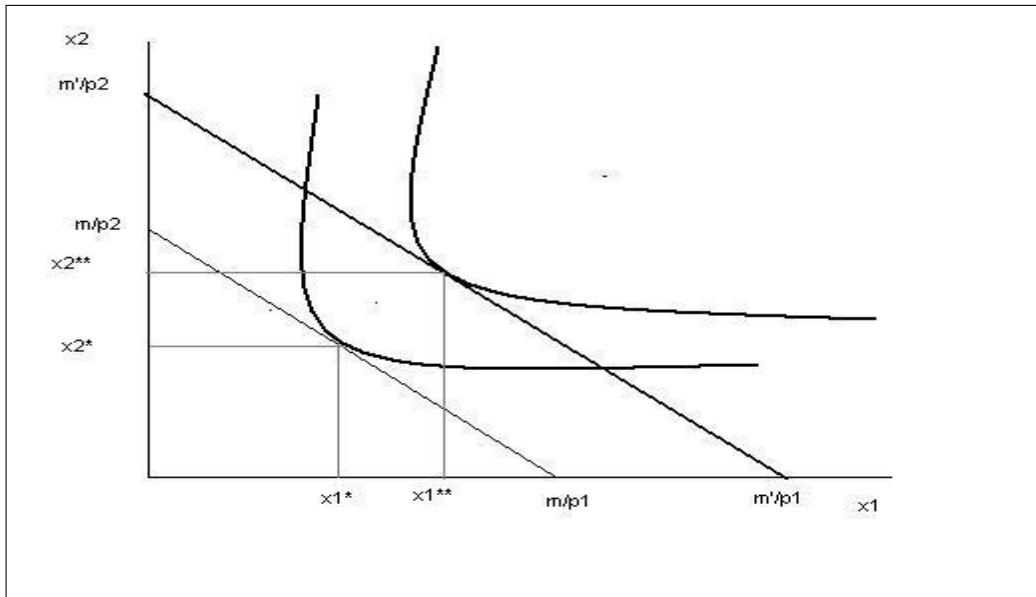
Therefore, the only equilibrium point is when  $\frac{MU_1}{p_1} = \frac{MU_2}{p_2}$ .

**Example 2** Consider  $U = 2x_1x_2$ . Plot for  $U = 100$ . Suppose  $p_1 = 1, p_2 = 2, m = 20$ . Plot budget line and identify optimal bundle.

### 1.3 Effect of Increase in Income on budget line

We know that when income increases, your budget lines shifts out. Given the new budget line, you can now attain a higher utility level. Therefore, you will reoptimize. Graphically, we see the intuitive result we would expect: with a higher income you can afford to consume more.

**Example 3** For the problem above, suppose  $m = 40$ . Plot and identify new optimal bundle.

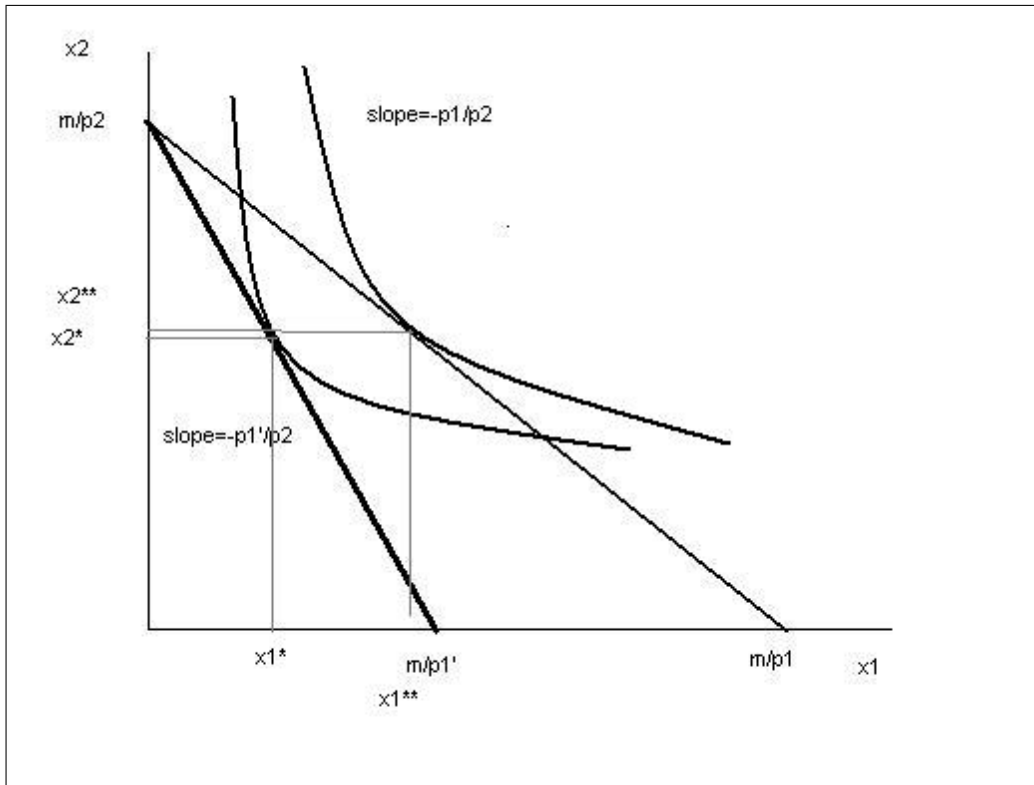


### 1.4 Effect of Decrease in $p_1$ on budget line

Suppose  $p_1$  increases but  $m$  and  $p_2$  don't change. We know that the budget constraint will shift out.

You can now attain a higher utility level. We can see graphically the effect. Not surprisingly, you now consume a bundle that involves more of the good which is cheaper in price. Your consumption of  $x_2$  has also increased slightly; this won't always be the case.

**Example 4** For the problem above, suppose  $m = 20$ ,  $p_1 = 2$ ,  $U = 50$ . Plot and identify new optimal bundle.



## 2 Perfect Substitutes Case

$$U = x_1 + x_2$$

Recall that the ic are straight lines and that so is the budget line.

Draw a picture and think through the logic.

3 possible cases

(1)  $p_2 > p_1$

recall, the slope of the budget line is  $-\frac{p_1}{p_2}$ . In this case, the slope of the budget line is flatter than

1.

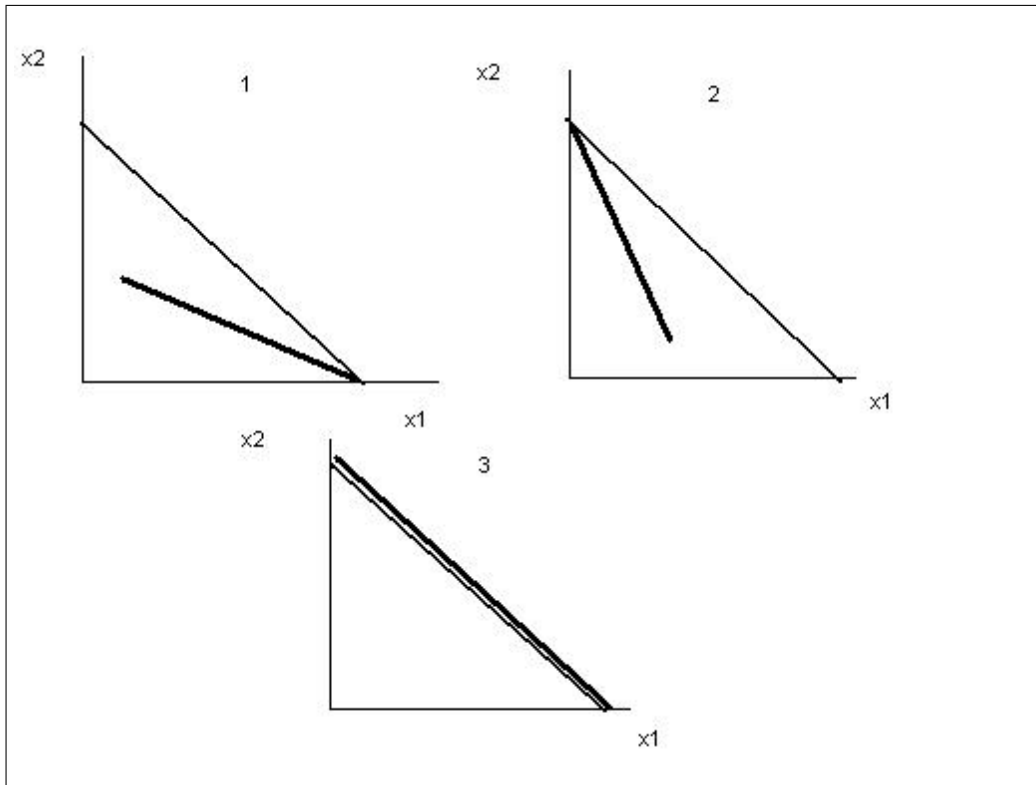
Calculate the slope of indifference curves.

$$x_2 = k - x_1$$

We can see that for this utility function, the slope of the indifference curve is 1. Therefore the budget line is flatter than the indifference curve. Graphically, can only intersect at one point, where  $x_2 = 0$  and spends all of income on  $x_1$ . Intuitively, since the goods are perfect substitutes, only buys cheaper good.

$$x_2 = 0 \rightarrow p_1 x_1 = m \rightarrow x_1 = \frac{m}{p_1}$$

(2)  $p_1 > p_2$



Slope of budget constraint is steeper than 1; therefore, slope of budget constraint is steeper than indifference curve. Buy no  $x_1$  and spend all of money on  $x_2$ .  $x_1 = \frac{m}{p_1}$

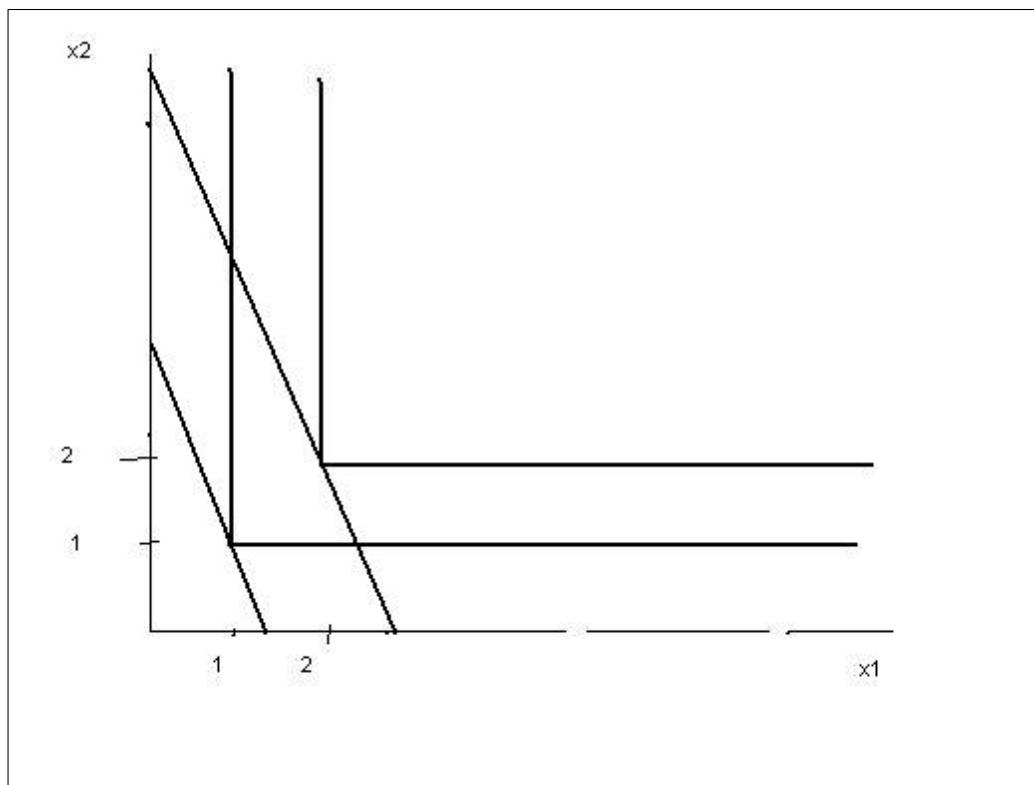
(3)  $p_1 = p_2$

Budget constraint and indifference curves have same slope. Will spend any number between 0 &  $\frac{m}{p_1}$  on  $x_1$  and any number between 0 &  $\frac{m}{p_1}$  on  $x_2$ .

**Example 5** Let  $U = 2x_1 + x_2$ ,  $m = 10$ ,  $p_1 = p_2 = 2$ . Draw the budget constraints, some indifference curves, identify the optimal bundle, and explain your result intuitively.

**Example 6** Suppose  $p_1$  increases to  $p_1 = 5$ . What happens?

### 3 Perfect Complements Case



Regardless of price ratio, always buy same amount of each good.

**Example 7** Let  $U = \min \{x_1, 2x_2\}$ ,  $p_1 = 1, p_2 = 2, m = 4$ . Plot and identify optimal bundle.